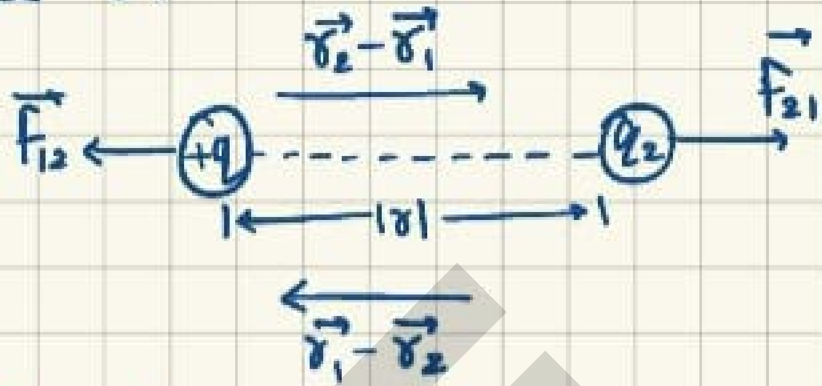
CHAPTER #1 : Electric Charges and Fields

Coulomb's Law of Electrostatics: (In vector form)

Consider two charges $+q$ and $-q$ separated by distance 'r'.
where, $\vec{F}_{12}$ = force exerted on q_1 by q_2
$\vec{F}_{21}$ = force exerted on q_2 by q_1



$$\therefore \vec{F}_{12} = \frac{kq_1q_2}{|r|^2}(\widehat{\vec{r}_1-\vec{r}_2})$$

$$= \frac{kq_1q_2}{|r|^2}\left[\frac{(\vec{r}_1-\vec{r}_2)}{|\vec{r}_1-\vec{r}_2|}\right] \quad \left(\because \hat{A}=\frac{\vec{A}}{|A|}\right)$$

$$= \frac{kq_1q_2}{|\vec{r}_1-\vec{r}_2|^2}\frac{(\vec{r}_1-\vec{r}_2)}{|\vec{r}_1-\vec{r}_2|} \quad (\because \vec{r}=\vec{r}_1-\vec{r}_2)$$

Aise boards में brackets में हर चीज का reason लिखना!

$$\therefore \boxed{\vec{F}_{12} = \frac{kq_1q_2}{|\vec{r}_1-\vec{r}_2|^3}(\vec{r}_1-\vec{r}_2)}$$ Hence Proved

Similarly, Force applied by $\vec{F}_{21}$, $\vec{F}_{21} = \frac{kq_1q_2}{|r|^2}(\widehat{\vec{r}_1-\vec{r}_2})$

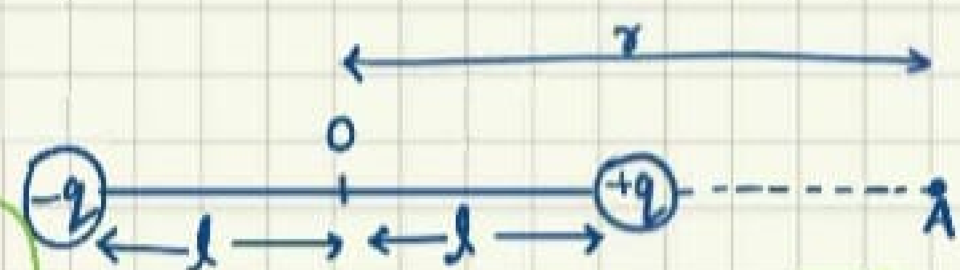
$$= \frac{kq_1q_2}{|r|^2}\left[\frac{(\vec{r}_2-\vec{r}_1)}{|\vec{r}_2-\vec{r}_1|}\right]$$

$$= \frac{kq_1q_2}{|\vec{r}_2-\vec{r}_1|^2}\frac{(\vec{r}_2-\vec{r}_1)}{|\vec{r}_2-\vec{r}_1|}$$

$$\therefore \boxed{\vec{F}_{21} = \frac{kq_1q_2}{|\vec{r}_2-\vec{r}_1|^3}(\vec{r}_2-\vec{r}_1)}$$ Hence Proved

Here, we can clearly observe that → $\vec{F}_{21} = -\vec{F}_{12}$, i.e. 3rd law of newton is valid in electrostatics also.

Electric Field due to dipole on axis:



We have two opposite charges separated by a distance of '$2l$', which makes it a dipole. A is a random point on axis at a distance of 'r' from centre of dipole.

इस तरह से थोड़ा 'gyaan' भी लिखो, teacher impress होगा।

Now, Field due to '$-q$' at A ⟶ $E_{-q} = \frac{-kq}{(r+l)^2}$

similarly, Field due to '$+q$' at A ⟶ $E_{+q} = \frac{kq}{(r-l)^2}$

So, Net Field ⇒ $E = E_{-q} + E_{+q}$

$$= \frac{-kq}{(r+l)^2} + \frac{kq}{(r-l)^2}$$

$$= \frac{-kq\,(r-l)^2 + kq\,(r+l)^2}{[(r+l)\,(r-l)]^2}$$

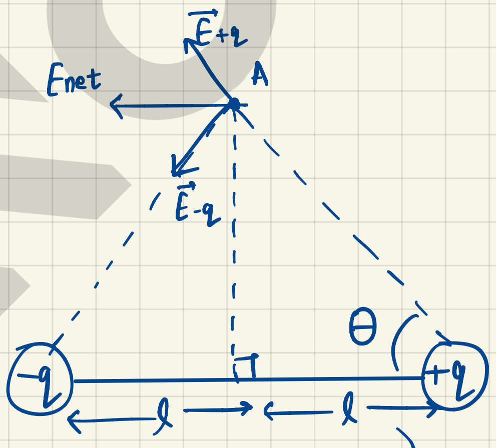
$$= \frac{2k\,(2ql)\,r}{(r^2-l^2)^2} \Rightarrow \frac{2kPr}{(r^2-l^2)^2}$$

Now, if $r >> l$, we can ignore l^2 in the denominator,

$$\vec{E} = \frac{2k\vec{p}\,\cancel{r}}{\cancel{r^4}}$$

$$\boxed{\vec{E} = \frac{2k\vec{p}}{r^3}}$$ Hence Proved

Electric field due to dipole on equitorial line:



$$E_{-q} = \frac{kq}{r^2+l^2} \qquad E_{+q} = \frac{kq}{r^2+l^2}$$

$$E_{net} = \sqrt{E_{-q}^2 + E_{+q}^2 + 2E_{-q}E_{+q}\cos 2\theta}$$

$$= \sqrt{\frac{k^2q^2}{(r^2+l^2)^2} + \frac{k^2q^2}{(r^2+l^2)^2} + \frac{2k^2q^2}{(r^2+l^2)^2}\cdot\cos 2\theta}$$

$$= \sqrt{\frac{2k^2q^2}{(r^2+l^2)^2} + \frac{2k^2q^2}{(r^2+l^2)^2}\cdot\cos 2\theta}$$

$$= \sqrt{\frac{2k^2q^2}{(r^2+l^2)^2}\left[1+\cos 2\theta\right]}$$

$$= \sqrt{\left(\frac{2kq}{(r^2+l^2)}\right)^2 \left[2\cos^2\theta\right]} \qquad (\because 1+\cos 2\theta = 2\cos^2\theta)$$

$$= \frac{2kq}{(r^2+l^2)}\,(\sqrt{2})\cos\theta$$

In this Δ,

Hypo = $\sqrt{r^2+l^2}$ (∵ pytha)

so, $\cos\theta = \frac{l}{\sqrt{r^2+l^2}}$ — (I)

putting cosθ here from (I),

$$E_{net} = \frac{-kP}{(r^2+l^2)^{3/2}}$$

If $r >> l$, l^2 can be neglected in the denominator.

Hence, $\boxed{E = \frac{-KP}{r^3}}$ Hence Proved

here -ve sign denotes the direction which is anti-parallel to direction of dipole (-ve to +ve)

Torque on dipole in external field:

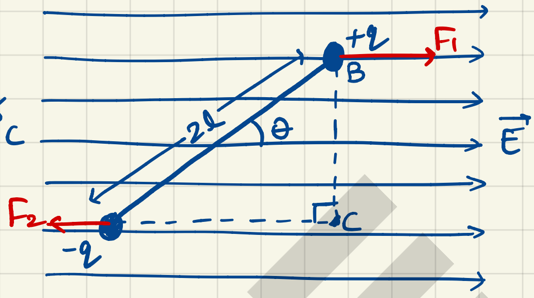


Figure shows an electric dipole with charges +q & -q at a separation of 2l placed in a uniform electric field ($\vec{E}$).

Dipole makes an angle θ with electric field.

$\vec{F_1} = -qE \longrightarrow$ force on charge -q

$\vec{F_2} = qE \longrightarrow$ force on charge q

$\vec{F_1} = -\vec{F_2}$

which means the force acting on dipole is equal in magnitude and opposite in direction at the two ends. Therefore it will behave like a couple.
As couple is acting on dipole, so it produces torque.

we know, $\tau = \begin{pmatrix}\text{magnitude of} \\ \text{either force}\end{pmatrix} \times \begin{pmatrix}\perp\text{r distance from} \\ \text{line of action of } F\end{pmatrix}$

$= F \times (BC)$

$= qE \times (2l \sin\theta)$

$$\boxed{\begin{aligned} \tau &= PE\sin\theta \\ \vec{\tau} &= \vec{P}\times\vec{E} \end{aligned}}$$ $(\because P = q(2l))$

Hence Proved

Case I : when $\theta = 0° \Rightarrow \therefore \sin 0 = 0$; which means $\boxed{\tau = 0}$.
this condition is called stable equilibrium because when the dipole is displaced from this orientation, it always comes back to same configuration.

Case II : when $\theta = 180° \Rightarrow \therefore \sin 180° = 0$; which means $\boxed{\tau = 0}$
this condition is called unstable equilibrium because once displaced the dipole never comes back to this orientation insted it aligns itself parallel to the field.

यहाँ हमने stable aur unstable को भी explain किया क्यूँकि हमें ऐसे सोचकर paper लिखना है कि teacher को कुछ नहीं आता। हमें सब समझाना होगा।

Case III : when $\theta = 90° \Rightarrow \therefore \sin 90° = 1$; which means τ is maximum.

$\tau = PE\cancel{\sin 90°}^{1}$

$\boxed{\tau = PE = \tau_{max}}$

Gauss law Verification using Coulomb's Law:

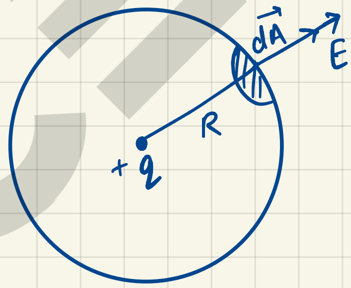
We know, the net electric field through a closed surface (3D) is $\frac{1}{\varepsilon_0}$ times the net charge enclosed by the surface.

$$\boxed{\phi_{closed} = \frac{q_{in}}{\varepsilon_0} = \oint \vec{E}\cdot d\vec{A}}$$

Verification:

According to electric flux,

$$\phi_E = \oint_s \vec{E}\,d\vec{s} = \oint_s E\,ds\cos\theta$$



We know, intensity of electric field $|\vec{E}|$ at same distance from charge q will remain constant,

also for spherical surface $\theta = 0°$

$\therefore$ electric flux :- $\phi_E = E\oint_s ds\cos 0°$

$$\phi_E = E\oint_s ds$$

$\left(\text{As } \oint_s ds \text{ means area} = 4\pi r^2\right)$

$$\therefore \quad \phi_E = E\,4\pi r^2 \quad \text{— (I)}$$

Now, according to coulomb's law, $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$

putting in (I), we get:-

$$\phi_E = \frac{1}{\cancel{4\pi}\varepsilon_0}\frac{q}{\cancel{r^2}} \times E\,\cancel{4\pi r^2}$$

$$\phi_E = \frac{1}{\varepsilon_0}q$$

or, $\boxed{\phi_E = \frac{1}{\varepsilon_0} \times (\text{enclosed charge})}$

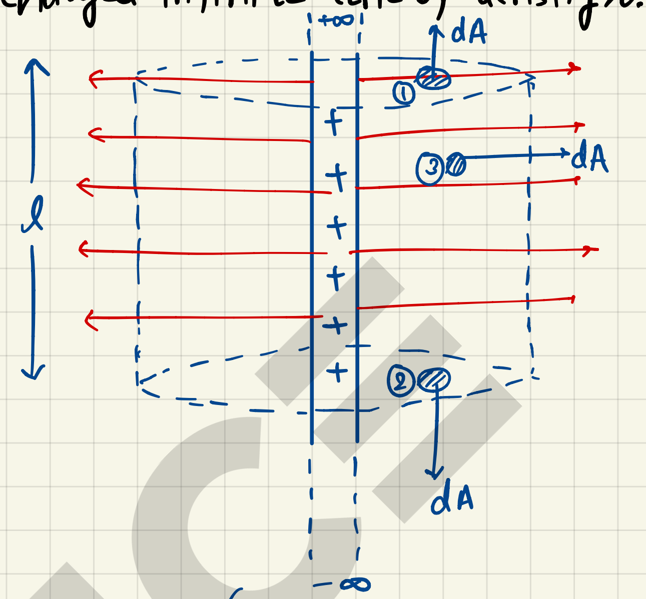
Hence Proved

Electric field due to a straight long charged conductor

or

Electric field due to a straight uniformly charged infinite line of density 'λ'.

Consider a part of length 'l' on this uniform conductor.
So, Gaussian surface will be cylindrical in this case.



Here, λ = linear charge den.
$\lambda = q/l$

Let dA be the small areas on this surface.
As conductor is positively charged, the direction of $\vec{E}$ field will be radially outwards.

Now,
$$\phi_1 = \int E\,dA\cos\theta \qquad [\theta = 90^\circ]$$
$$\phi_2 = \int E\,dA\cos\theta \qquad [\theta = 90^\circ]$$
$$\phi_3 = \int E\,dA\cos\theta \qquad [\theta = 0^\circ]$$

Hence, Net flux :- $\phi_{net} = \phi_1 + \phi_2 + \phi_3$

$$= \int E\,dA\cos 90^\circ + \int E\,dA\cos 90^\circ + \int E\,dA\cos 0^\circ$$
$$= 0 + 0 + \int E\,dA$$
$$\phi = EA$$

$$\therefore \boxed{\phi = E(2\pi r l)} \quad \text{(I)} \qquad [\because \text{Total curved area of surface} = 2\pi r l]$$

Now, Acc. to Gauss Law :

$$\phi = \frac{q_{in}}{\varepsilon_0}$$

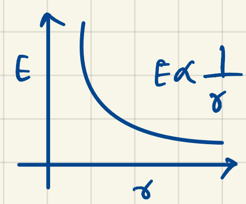
from (I),
$$E(2\pi r \cancel{l}) = \frac{\lambda \cancel{l}}{\varepsilon_0} \qquad [\because q_{in} = \lambda l]$$

$$\boxed{E = \frac{\lambda}{2\pi r \varepsilon_0}}$$

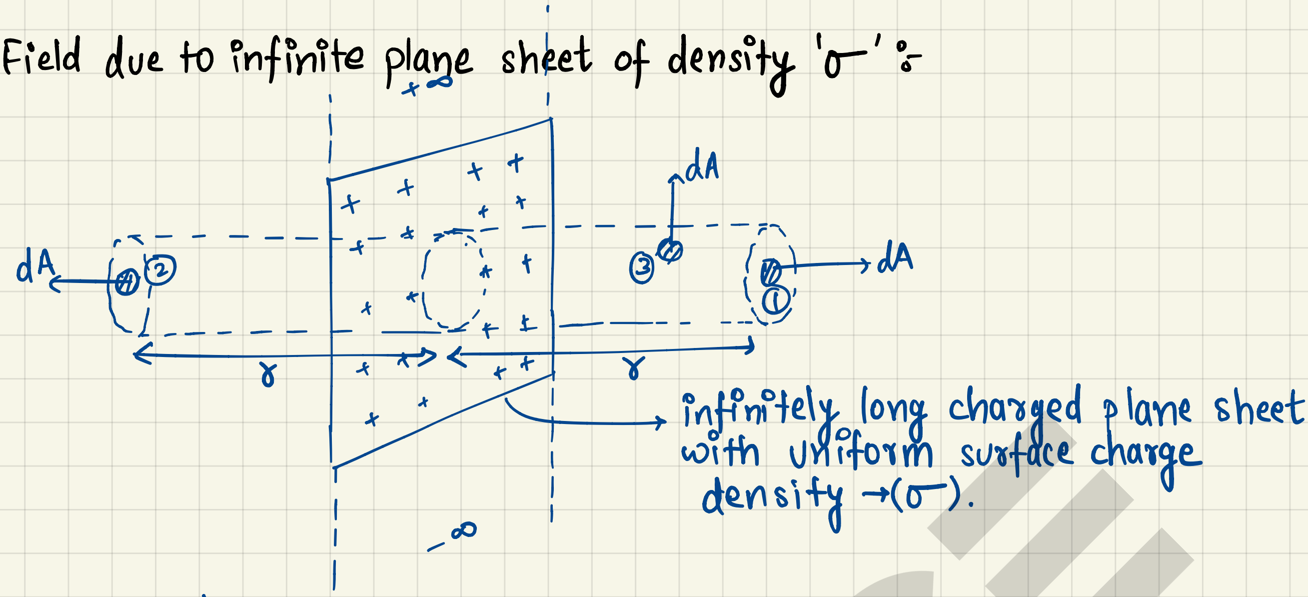
Hence Proved

Here, we can clearly see, $E \propto \frac{1}{r}$

So, Graphically :-



Electric Field due to infinite plane sheet of density 'σ':



Draw a Gaussian cylinder of area of radius 'r' . Take 3 sample small surfaces 'dA' at ①, ② & ③.

Total flux, $\phi_{net} = \phi_1 + \phi_2 + \phi_3$

$$= \int E\,dA \cos 0^\circ + \int E\,dA \cos 0^\circ + \int E\,dA \cos 90^\circ$$

$$= \int E\,dA + \int E\,dA + 0$$

$$= EA + EA$$

$$\boxed{\phi = 2EA} \leftarrow \text{I}$$

Acc to gauss law, $\phi = \dfrac{q_{in}}{\varepsilon_0} \Rightarrow \dfrac{\sigma A}{\varepsilon_0}$

from I & II $\Rightarrow$ $2EA = \dfrac{\sigma A}{\varepsilon_0}$

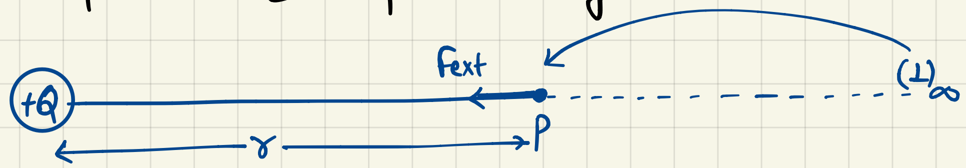
$$\boxed{E = \frac{\sigma}{2\varepsilon_0}}$$

independent of r

Hence Proved

CHAPTER #2: Electric Potential and Capacitance

Potential at a point due to point charge:



Let there be a point P at a distance 'r' from +Q charge.
Electric potential means workdone to bring a unit +ve charge from infinite to the point P.

$$\therefore W_{(P\to\infty)} = \int_P^{\infty} F_{ext}\, dr$$

$$= \int_P^{\infty} \frac{kQ(1)}{r^2}\, dr \cos\theta$$

$$= -kQ \int_P^{\infty} \frac{1}{r^2}\, dr \qquad [\because \theta = 180^\circ \Rightarrow \cos 180^\circ = -1]$$

$$= -kQ \left[\frac{-1}{r}\right]_r^{\infty}$$

$$= -kQ \left[-\frac{1}{\infty} - \left(-\frac{1}{r}\right)\right]$$

$$= -kQ \left[-\frac{1}{\infty} + \frac{1}{r}\right]$$

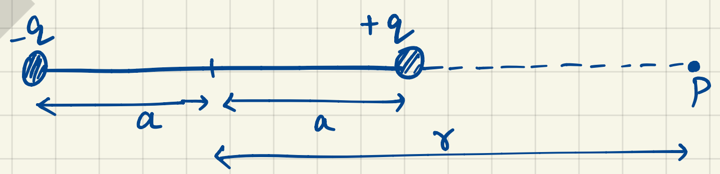
$$W_{(P\to\infty)} = \frac{KQ}{r}$$

or,
$$\boxed{V = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r}}$$

Hence Proved

Potential due to dipole:

(a) At a point on axial line:



Consider a dipole with charges +q & -q separated by a distance of '2a'. A point P lies on the axial line at a distance 'r' from centre of dipole.

So, Potential at P due to +q, $V_+ = \frac{kq}{r-a}$

due to -q, $V_- = \frac{-kq}{r+a}$

$\therefore$ Net potential at P, $V_{axial} = (V_+) + (V_-)$

$$= \left(\frac{kq}{r-a}\right) + \left(\frac{-kq}{r+a}\right)$$

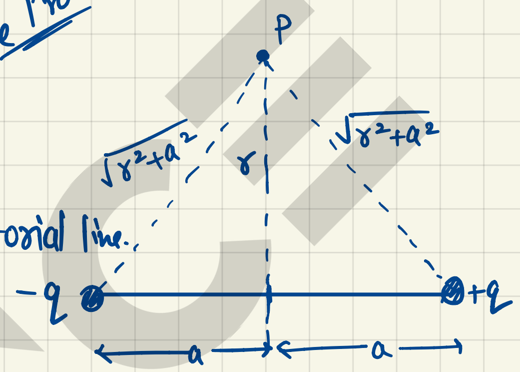
$$= \frac{kq(r+a) - kq(r-a)}{r^2 - a^2}$$

$$= \frac{(2aq)k}{r^2 - a^2}$$

Hence, $V_{axial} = \dfrac{kp}{r^2 - a^2}$

for short dipole $(r >> a)$ → $\boxed{V_{axial} = \dfrac{kp}{r^2}}$ Hence Proved

(b) At a point on Equatorial line:



Let there be a point 'P' at a distance 'r' on equatorial line.

So, as in diagram:

Potential at P due to +q, $V_+ = \dfrac{kq}{\sqrt{a^2+r^2}}$

due to −q, $V_- = \dfrac{k(-q)}{\sqrt{a^2+r^2}}$

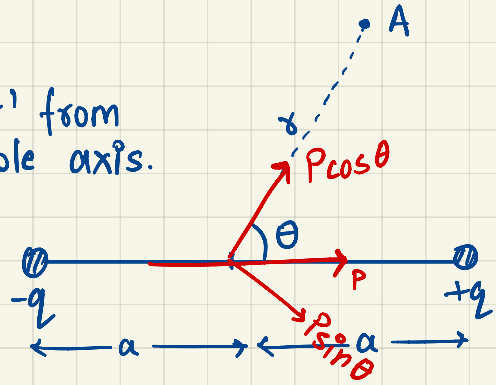
So, Net potential at P, $V_{eq} = (V_+) + (V_-)$

$$= \frac{kq}{\sqrt{a^2+r^2}} + \left(\frac{-kq}{\sqrt{a^2+r^2}}\right)$$

$\boxed{V_{eq} = 0}$ Hence Prooved

Hence, electric potential due to dipole at any point on eq. line will be 0.

(c) At any arbitrary point:



let A be any arbitrary point at a distance 'r' from centre of dipole making an angle θ with dipole axis.

Observe the figure carefully: If we resolve dipole moment($\vec{p}$) into two rectangular components as shown.

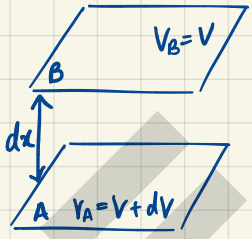
Then, point A lies on axial line of dipole with dipole moment '$p\cos\theta$'
so, potential at a due to this component $= \dfrac{k(p\cos\theta)}{r^2}$

and point A lies on equatorial line of dipole with dipole moment '$p\sin\theta$'
but as discussed above, as A is on eq. line ∴ potential due to this component will be zero.

Hence, $$V_{net} = \frac{kp\cos\theta}{r^2} + 0$$

$$\boxed{V = \frac{kp\cos\theta}{r^2}}$$ Hence Proved

Relation between Electric field and Potential:



Consider two equipotential surface A and B separated by a distance of 'dx', let the potential of surface B be $V_B = V$
and of A be $V_A = V + dV$

Now, work done to displace unit positive charge from B to A:

$$dW = \vec{F}\,dx\cos 180^\circ$$
$$dW = -\vec{F}\,dx$$

$$\left[\text{As} \quad E = \frac{F}{\cancel{q}\to 1} \qquad \therefore E = F\right]$$

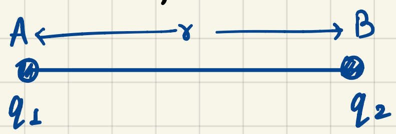
So, $dW = -\vec{E}\,dx$ — (I)

Also, we know $dW = q(V_A - V_B)$
$$dW = (1)(\cancel{V} + dV - \cancel{V})$$
$$dW = dV \quad \text{— (II)}$$

from (I) & (II) $\Rightarrow -\vec{E}\,dx = dV$

$$-\vec{E} = \frac{dV}{dx} \Rightarrow \boxed{E = \frac{-dV}{dx}}$$ Hence Proved

Potential Energy of System of two point charge (in absence of E.F.)



Initially there were no charge at A and B.

Firstly, we'll bring q_1 from ∞ to A.
So, work done to place charge q_1 to A, $W_A = q_1 V_A$
$= 0$ $[\because V_A = 0$, i.e. potential energy of static charge$]$

Now, we'll bring q_2 from ∞ to B (and in this case q_1 is already at A)

So, potential at B due to q_1 at A, $V_B = \frac{kq_1}{r}$ — (I)

$\therefore$ Work done to place q_2 at B, $W_B = q_2 V_B$
$$= q_2\left(\frac{kq_1}{r}\right) \quad [\text{from (I)}]$$
$$W_B = \frac{kq_1q_2}{r}$$

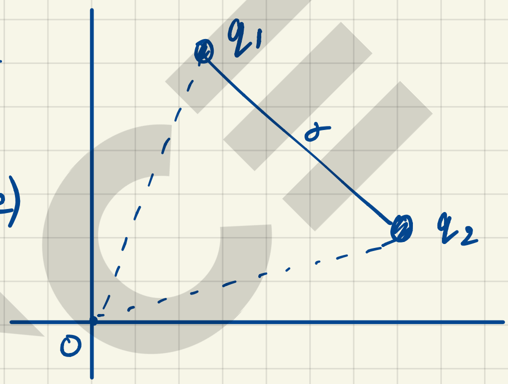
And as we know, sum of work done is equal to the potential energy of system

$\therefore$ Potential Energy$(U) = 0 + \frac{kq_1q_2}{r}$

$$\boxed{U = \frac{kq_1q_2}{r}}$$ Hence Proved

Potential Energy of a system of two charges in an external electric field :-

Let potential at A and B be V_A and V_B respectively.



Now, Work done to place q_1 at A

$W_A = q_1 V_A$ —(I) ($\because$ initially q_2 was not there)

Work done to place q_2 at B

$W_B = q_2 V_B + \frac{kq_1q_2}{r}$ —(II)

$\therefore$ Net work done, $W = W_A + W_B$

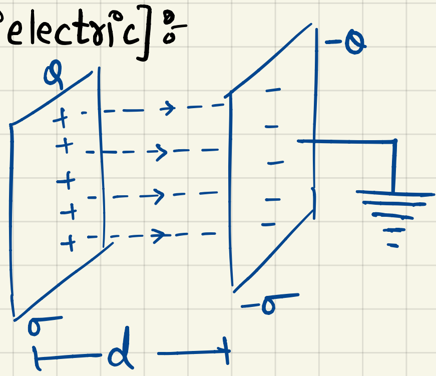
$= q_1V_A + q_2V_B + \frac{kq_1q_2}{r}$

And as we know, this work done is equal to potential energy of system.

$$\boxed{U = q_1V_A + q_2V_B + \frac{kq_1q_2}{r}}$$ Hence Proved

Capacitance of a parallel plate capacitor [without dielectric] :-

Consider a parallel plate capacitor of plate area A, and separation d.



Let $\pm\sigma$ be the surface charge density.
Electric field outside capacitor plates is zero.

Now, the electric field inside the capacitor plate,

$\vec{E} = \frac{\sigma}{\varepsilon_0}$ (by application of gauss law of charge plate)

Potential difference between the plates, $V = \vec{E}d$

$\Rightarrow V = \frac{\sigma}{\varepsilon_0}d$

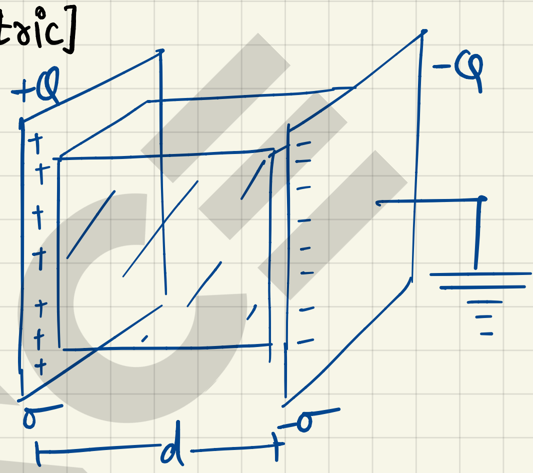
$\Rightarrow V = \frac{Q}{A\varepsilon_0}d$ —(I)

We know that, $C = \frac{Q}{V}$

$$\Rightarrow C = \frac{\cancel{Q}}{\frac{\cancel{Q}}{A\varepsilon_0} d} \quad (\text{from } ②)$$

$$\Rightarrow \boxed{C = \frac{A\varepsilon_0}{d}}$$ Hence Proved

Capacitance of parallel plate capacitor [with dielectric]



Consider a parallel plate capacitor of plate area 'A' and separation d.

Let $\pm\sigma$ be the surface charge density. The gap between the plate is filled with dielectric substance having dielectric constant K.

The electric field between plates will be:

$$E = \frac{\sigma}{\varepsilon_0 K}$$

$$E = \frac{Q}{A\varepsilon_0 K} \quad \text{— ①} \quad \left(\because \sigma = \frac{Q}{A}\right)$$

$\therefore$ Potential difference between the plate

$$V = Ed$$

$$\rightarrow V = \frac{Q}{A\varepsilon_0 K} d \quad \text{— ②} \quad (\text{Using ①})$$

Now, Capacitance, $C' = \frac{Q}{V}$

$$C' = \frac{\cancel{Q}}{\frac{\cancel{Q}}{A\varepsilon_0 K} d} \quad (\text{from ②})$$

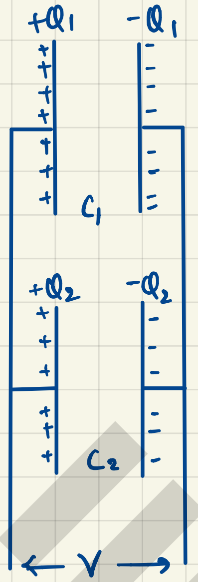
$$C' = \frac{A\varepsilon_0 K}{d}$$

or, $\boxed{C' = CK}$ or $\boxed{C' = KC}$

where, C' = capacitance with dielectric
C = capacitance without dielectric
K = dielectric constant of the medium.

Conclusion: After inserting dielectric medium in between the plates of a capacitor, it's capacitance increases by 'K' times initial capacitance

Capacitance in Parallel:

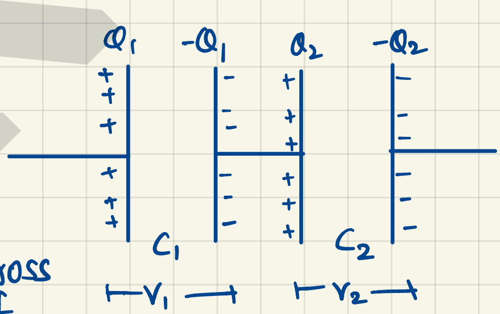


Consider two capacitors connected in parallel combination as shown in the figure
In parallel combination potential difference across all the capacitors remains same but distribution of charge across each capacitor will be different.

$$\therefore \quad Q = Q_1 + Q_2$$
$$\Rightarrow CV = C_1V + C_2V \qquad (\because Q = CV)$$
$$C\cancel{V} = \cancel{V}(C_1 + C_2)$$
$$\Rightarrow \boxed{C = C_1 + C_2}$$ Hence Proved

The effective capacitance of a combination of 'n' capacitors in parallel combination is algebric sum of capacitance of each capacitors.

Capacitors in Series:



Consider two capacitors are connected in series combination in a circuit with capacitance C_1 and C_2 respectively as shown in figure.

In series combination, the potential difference across each capacitor is different but distribution of charge remains same.

$$\therefore \quad V = V_1 + V_2$$
$$\Rightarrow \frac{Q}{C} = \frac{Q}{C_1} + \frac{Q}{C_2} \Rightarrow \left(\frac{1}{C}\right)\cancel{Q} = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)\cancel{Q}$$
$$\boxed{\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}}$$

Which means, the effective capacitance of a combination of 'n' capacitors in series is:

$$\boxed{\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \text{---} + \frac{1}{C_n}}$$

Energy Stored in Capacitor AND Expression for Energy density:

Let 'dq' be the small amount of charge transferred by the source from the plate to -ve plate.

$\therefore$ Then work done by the source is $dW = V\,dq \qquad \left[\because V = \frac{dW}{dq}\right]$

$$\Rightarrow dW = \frac{q}{C}dq \qquad [\because q = CV]$$

$\therefore$ Total work done by the source is transferring amount of charge.

$$W = \int dw$$

$$\Rightarrow W = \int \frac{q}{C} dq$$

$$\Rightarrow W = \frac{1}{C} \int q \, dq$$

$$\Rightarrow W = \frac{1}{C} \left(\frac{q^2}{2} \right)$$

$$\Rightarrow W = \frac{1}{2} \frac{q^2}{C}$$

$$\Rightarrow W = \frac{1}{2} \frac{(CV)^2}{C} \quad [\because q = CV]$$

$$\Rightarrow W = \frac{1}{2} \frac{C^{\cancel{2}} V^2}{\cancel{C}}$$

$$\Rightarrow \boxed{W = \frac{1}{2} CV^2}$$

Now, the work done is in the form of potential energy, i.e. :-

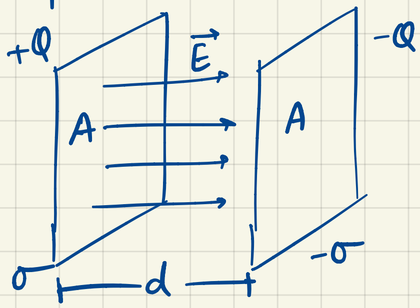
$$\boxed{U = \frac{1}{2} CV^2}$$

or,
$$U = \frac{1}{2} \frac{q}{\cancel{V}} V^{\cancel{2}}$$

$$\Rightarrow \boxed{U = \frac{1}{2} qV}$$ Hence Proved

ENERGY DENSITY :-

The potential energy per unit volume of a capacitor is known as Energy density.



$\therefore$ Energy density, $\mu = \dfrac{U}{\text{volume}}$

$$\mu = \frac{\frac{1}{2} CV^2}{Ad}$$

$$= \frac{\frac{1}{2} \frac{Q^2}{C}}{A \times d}$$

$$= \frac{\frac{1}{2} \frac{(EA \, \varepsilon_0)^2}{\frac{\varepsilon_0 A}{d}}}{A \times d}$$

$$= \frac{1}{2} \frac{E^2 A^2 \varepsilon_0^2 \times d}{A^2 \times d \, \varepsilon_0 A}$$

$$\boxed{u = \frac{1}{2} \varepsilon_0 E^2}$$

If any medium is there between plates of a capacitor,

$$\boxed{u = \frac{1}{2} \varepsilon_0 \varepsilon_r E^2}$$

CHAPTER #3 : Current Electricity

Obtain an expression for Drift Velocity of Electrons :-

Drift velocity is the velocity with which electrons in a conductor are drifted towards the positive terminals of the potential source.

We know that in a conductor there are N number of electrons.

Initially, without any electric field, the electrons in the conductor move randomly with some velocity (V_i)

i.e. $$\frac{1}{N}\sum_{i=1}^{N} V_i = 0 \quad \text{—(I)}$$

Now, when an electric field is applied across the conductor; The force applied on a electron by the electric field is :-

$$F = -eE \qquad [\because F = qE]$$
$$\Rightarrow ma = -eE \qquad [\because F = ma]$$
$$\Rightarrow a = \frac{-eE}{m} \quad \text{—(II)}$$

where, a = acceleration of $e^{\ominus}$ towards +ve terminal.
m = mass of the electron.

If we take 'τ' to be the average relaxation time (the time interval between any two successive collision)

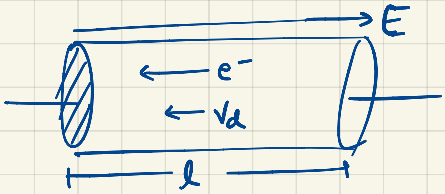
then by first equation of motion,

$$(V_i)_{avg} = (U_i)_{avg} + a(t_i)_{avg}$$
$$V_d = 0 + \left(\frac{-e\vec{E}}{m}\right)(\tau)$$
$$\Rightarrow \boxed{V_d = \frac{-e\vec{E}\tau}{m}}$$

where $V_d = (V_i)_{avg}$ = drift velocity

Hence Proved

Relation between current and drift velocity :-



Consider a conductor of length l and area of cross-section 'A' and 'n' be the no. of $e^{\ominus}$ present per unit volume.

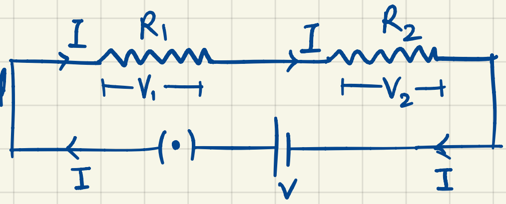
$\therefore N = nAl$

Total charge, $Q = nAle$

$\therefore$ current in the conductor,

$$I = \frac{Q}{t}$$
$$= \frac{nAle}{t}$$
$$\boxed{I = neAV_d} \quad \left(\because V_d = \frac{l}{t}\right)$$

Series combination of Resistance:



Two resistors of resistance R_1 and R_2 are connected in series.
As we know, in series combination, current is same but voltage is different across the components

$$\therefore \quad V = V_1 + V_2$$

using ohm's law,

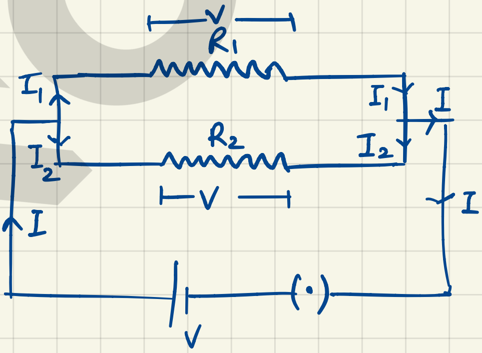
$$IR = IR_1 + IR_2$$

$$\cancel{I}R = \cancel{I}(R_1 + R_2)$$

$$\boxed{R = R_1 + R_2}$$ Hence Proved

$\therefore$ for 'n' no. of resistance in series, $R = R_1 + R_2 + R_3 + \text{------} R_n$.

Parallel combination of Resistance:



Two resistors R_1 and R_2 are connected in parallel with a battery of voltage 'V'.

As we know, in parallel combination voltage remains same and current is different across the components of the circuit.

$$\therefore \quad I = I_1 + I_2$$

using ohm's law,

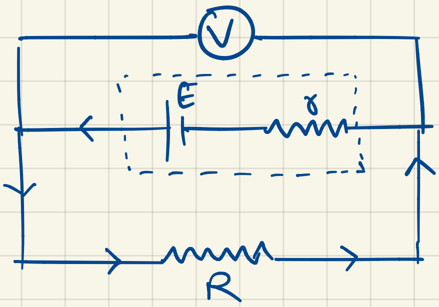
$$\frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2}$$

$$\Rightarrow \cancel{V}\left(\frac{1}{R}\right) = \cancel{V}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$\boxed{\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}}$$ Hence Proved

$\therefore$ for 'n' resistors in parallel, $\dfrac{1}{R} = \dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3} + \text{------} + \dfrac{1}{R_n}$

Relation between Internal resistance, terminal potential diff. and EMF:-



Consider a cell of emf 'E' with internal resistance 'r' connected to the external resistance (R). The current in the circuit is:

$$I = \frac{E}{R+r} \quad \text{— (I)} \qquad \left[\because I = \frac{\text{total EMF}}{\text{total resistance}}\right]$$

Terminal potential difference

$$V = IR \quad \text{— (II)}$$

Now, (I) can be written as $\rightarrow$ $I(R+r) = E$ (cross-multiply)

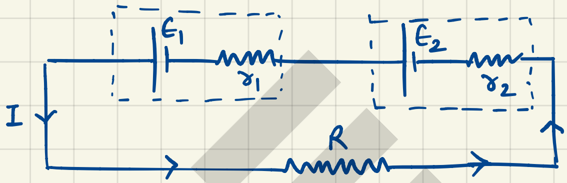
$IR + Ir = E$

$V + Ir = E$ (from (II))

$\boxed{V = E - Ir}$ (For $V<E$)

for $V>E$ $\boxed{V = E + Ir}$ which is the relation b/w E, V and r.

Cells in series :



Consider two cells with emf E_1 and E_2 and having internal resistance r_1 and r_2 respectively connected in series.

$V_1 = E_1 - Ir_1$ (for $V<E$)

also, $V_2 = E_2 - Ir_2$

We know, in series current is same but potential across components is diff.

$\therefore$ $V_{eq} = V_1 + V_2$

$V_{eq} = (E_1 - Ir_1) + (E_2 - Ir_2)$

$V_{eq} = (E_1 + E_2) - (Ir_1 + Ir_2)$

$V_{eq} = (E_1 + E_2) - (r_1 + r_2)I$ —— (I)

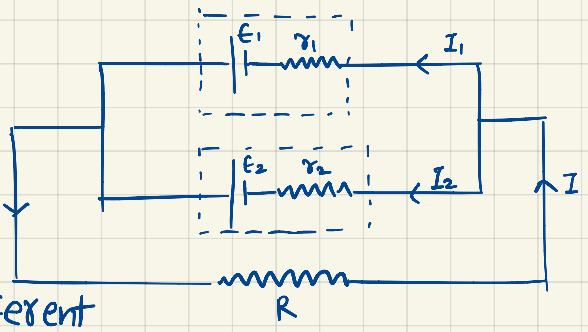
Now, we know $V_{eq} = E_{eq} - Ir_{eq}$ —— (II)

Comparing (I) & (II),

$$\boxed{\begin{aligned} E_{eq} &= E_1 + E_2 + \text{-----} \\ r_{eq} &= r_1 + r_2 + \text{-----} \end{aligned}}$$

Hence Proved

Cells in Parallel :



Consider two cells of emf E_1 and E_2 with internal resistance r_1 and r_2 respectively, connected in parallel

We know, in parallel combination potential diff is same but current will be different across components.

$\therefore$ $I = I_1 + I_2$

$I = \dfrac{E_1 - V}{r_1} + \dfrac{E_1 - V}{r_2}$ $\left[\because V = E - Ir \Rightarrow I = \dfrac{E - I}{r}\right]$

$I = \dfrac{E_1}{r_1} + \dfrac{E_2}{r_2} - V\left(\dfrac{1}{r_1} + \dfrac{1}{r_2}\right)$

$\Rightarrow V\left(\dfrac{r_1 + r_2}{r_1 r_2}\right) = \dfrac{E_1 r_2 + E_2 r_1}{r_1 r_2} - I$

$$\Rightarrow \quad V = \frac{E_1 r_2 + E_2 r_1}{r_1 + r_2} - I\left(\frac{r_1 r_2}{r_1 + r_2}\right)$$

Comparing this with $\quad V = E_{eq} - I r_{eq}$

$$\boxed{E_{eq} = \frac{E_1 r_2 + E_2 r_1}{r_1 + r_2}}$$

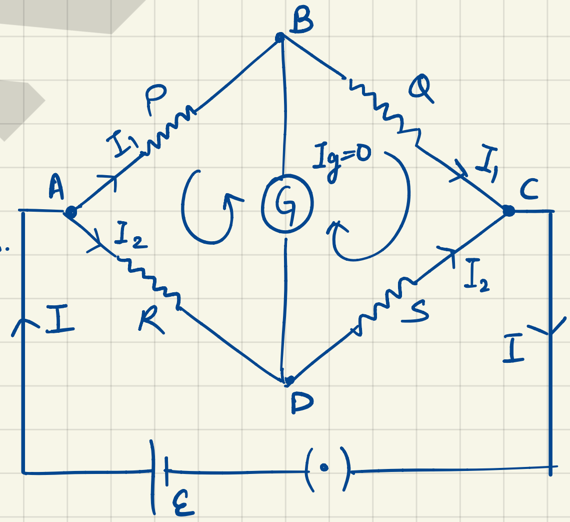
and, $$\boxed{r_{eq} = \frac{r_1 r_2}{r_1 + r_2}}$$ *Hence Proved*

We can also write these equations in simple ways,

$$\boxed{\frac{E_s}{r_{eq}} = \frac{E_1}{r_1} + \frac{E_2}{r_2} + \cdots}$$

and, $$\boxed{r_{eq} = \frac{1}{r_1} + \frac{1}{r_2} + \cdots}$$

Wheatstone Bridge :



Wheatstone bridge is an arrangement of four resistance used to determine resistance of one resistors in terms of other three resistors.

For a balanced bridge,
$V_B = V_D$ (as in figure)

Now, applying Kirchoffs rule on loop ADBA :-

$$I_2 - R - I_1 P = 0$$
$$\Rightarrow \quad I_1 P = I_2 R \quad \text{—— (I)}$$

Now, applying Kirchoffs rule on loop BCDB :-

$$I_1 Q - I_2 S = 0$$
$$\Rightarrow \quad I_1 Q = I_2 S \quad \text{—— (II)}$$

dividing eq (I) & (II) ; we get : $\quad \frac{(I)}{(II)} \Rightarrow \frac{I_1 P}{I_1 Q} = \frac{I_2 R}{I_2 S}$

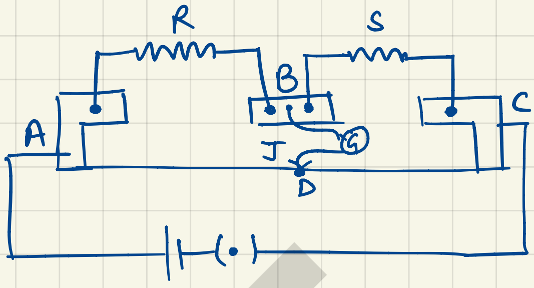
$$\Rightarrow \quad \boxed{\frac{P}{Q} = \frac{R}{S}}$$ *Hence Proved*

This is the condition for balanced wheatstone bridge.

Finding unknown resistance using slide wire bridge:

OR

Principle of meterbridge and finding unknown resistance



Principle : Wheat stone bridge

As shown in figure,

R= unknown resistance
S = known resistance

Move the jockey (J) on wire AC of length l to obtain the null point (i.e. zero reading of the galvanometer). Let point D be null point on wire AC.

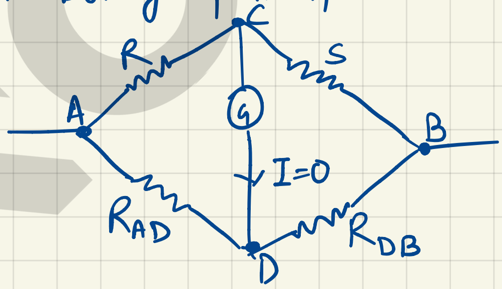
As the bridge is balanced, therefore, by wheatsone bridge principle :-

$$\frac{R}{R_{AD}} = \frac{S}{R_{DB}}$$

$$\frac{R}{r l} = \frac{S}{r(100-l)}$$

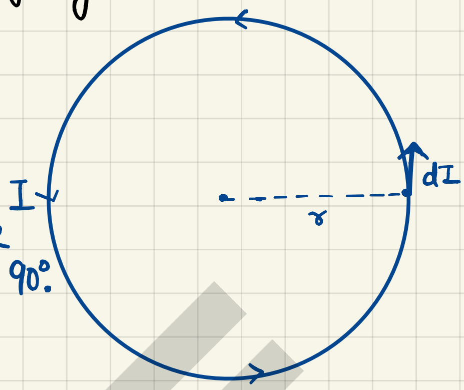
$$\boxed{R = \frac{S l}{100-l}}$$

Hence Proved



CHAPTER #4 : Moving Charges and Magnetism

Magnetic field at the centre of a circular loop carrying current :



Consider a circular current carrying loop carrying current I. We have to find magnetic field at the centre of this loop.

Consider a small current element d on circumference of this loop. Clearly angle between d and r is 90°.

Applying Biot Savart's law, we get

$$dB = \frac{\mu_0}{4\pi}\left(\frac{I\,dl\,\sin 90^\circ}{r^2}\right)$$

$$\Rightarrow dB = \frac{\mu_0}{4\pi}\frac{I\,dl}{r^2}$$

integrating both sides, we get :

$$\int dB = \int \frac{\mu_0}{4\pi}\frac{I\,dl}{r^2}$$

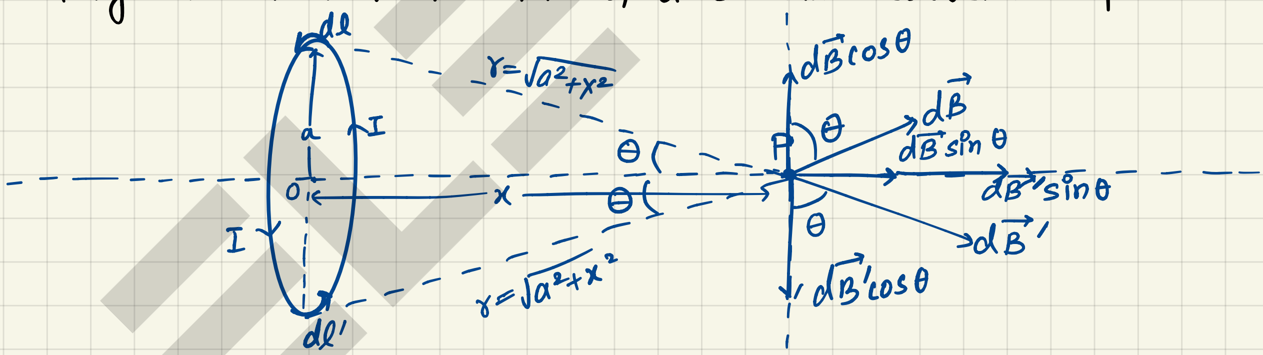
$$\Rightarrow B = \frac{\mu_0}{4\pi}\frac{I}{r^2}\int dl$$

$$\Rightarrow B = \frac{\mu_0}{4\pi}\frac{1}{r^2}\times 2\pi r \qquad \left(\because \int dl \text{ means total circumference}\right)$$

$$\Rightarrow \boxed{B = \frac{\mu_0 I}{2r}}$$

Hence Proved

Magnetic Field on the axis of a circular current loop :-



Consider a circular loop of radius 'a' the axis of the circular loop at which we have to calculate the magnetic field due to the circular loop and x is the distance between the loop and the point 'P'.

According to Biot Savarts law,

$$dB = \frac{\mu_0}{4\pi}\frac{I\,dl\,\sin\theta}{r^2}$$

So, the magnetic field at P due to current element $I\vec{dl}$:

$$dB = \frac{\mu_0}{4\pi} \frac{Idl \sin 90^\circ}{r^2} \quad [\because r \perp dl]$$

$$\Rightarrow dB = \frac{\mu_0}{4\pi} \frac{Idl}{(a^2+x^2)}$$

Magnetic field at P due to current element $I\vec{dl}$

$$dB' = \frac{\mu_0}{4\pi} \frac{Idl \sin 90^\circ}{r^2}$$

$$\Rightarrow dB' = \frac{\mu_0}{4\pi} \frac{Idl'}{(a^2+x^2)}$$

we can see → Here, $dB = dB'$

Resolving dB in two components, we find that $\cos\theta$ component for two diametrically opposite elements cancel each other.
So, that magnetic field intensity at P will be only due to $\sin\theta$ component

There fore, total magnetic field due to the whole coil.

$$\vec{B} = \int dB \sin\theta$$

$$\vec{B} = \int \frac{\mu_0}{4\pi} \frac{Idl \sin\theta}{a^2+x^2}$$

$$\vec{B} = \frac{\mu_0 I \sin\theta}{4\pi (a^2+x^2)} \int dl$$

$$\vec{B} = \frac{\mu_0 I}{4\pi (a^2+x^2)} \frac{a}{\sqrt{a^2+x^2}} \int dl$$

$$\vec{B} = \frac{\mu_0 I a}{\cancel{4\pi}_{2} (a^2+x^2)(a^2+x^2)^{1/2}} \quad \cancel{2\pi} a$$

$$\boxed{\vec{B} = \frac{\mu_0 I a^2}{2(a^2+x^2)^{3/2}}}$$

Hence Proved

if $x >>> a$, then a is neglegible

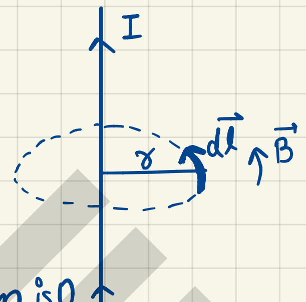
$$\vec{B} = \frac{\mu_0 I}{2(x^{\cancel{2}})^{3/\cancel{2}}}$$

$$\boxed{\vec{B} = \frac{\mu_0 I}{2x^3}}$$

Ampere's Circuital Law:

It states that the line integral of magnetic field intensity over a closed loop is μ_0 times the total current threading the loop.

↳ i.e. $\oint \vec{B}\cdot d\vec{l} = \mu_0 I$



Proof:- Consider a straight conductor carrying as shown in the figure. Consider a circular American loop of radius r around the conductor.

As $\vec{B}$ and $d\vec{l}$ are in same direction so angle between them is 0.

$\therefore$

$$\int \vec{B}\cdot d\vec{l}$$
$$= \int B\,dl \cos 0^\circ$$
$$= \int B\,dl$$
$$= B\int dl$$
$$= \frac{\mu_0 I}{\cancel{2\pi r}} \times \cancel{2\pi r}$$

$(\because \int dl$ means circumference $= 2\pi r)$

$$\boxed{\int B\,dl = \mu_0 I}$$

Hence Prooved

Application of Ampere's Circuital Law
- → M.F. due to ∞ long straight current carrying conductor
- → Solenoid
- → Toroid

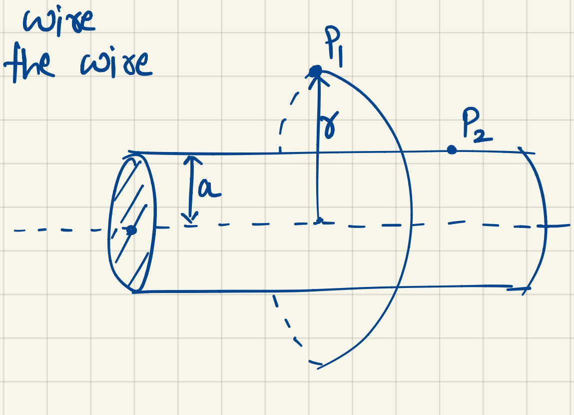
Magnetic field due to an infinitely long straight current carrying conductor:

We've given a long straight wire of a cross-sectional radius 'a' carrying steady current I. This current is uniformly distributed across this cross-section.

Now, we have to calculate magnetic field at a distance r from centre. But here we'll have 3 cases:-

(i) $r > a$; i.e. point lies outside wire
(ii) $r = a$; i.e. point lies on the wire
(iii) $r < a$; i.e. point lies inside the wire

CASE (I):- $r > a$ at point P_1.



Now, to find the magnetic field at point P_1 outside the wire → make a circular loop made of radius 'r' as shown in figure.

Using Ampere's Law,

$$\Rightarrow \oint \vec{B}\cdot\vec{dl} = \mu_0 I$$

$$\Rightarrow \oint B\,dl \cos 0^\circ = \mu_0 I$$

$$\Rightarrow B \oint dl = \mu_0 I$$

$$\Rightarrow B(2\pi r) = \mu_0 I \quad (\because \oint dl \text{ means circumference} = 2\pi r)$$

$$\boxed{B = \frac{\mu_0 I}{2\pi r}} \text{ — (I)}$$

[where r is the distance of point from centre]

$$B \propto \frac{1}{r} \quad (\text{for } r > a)$$

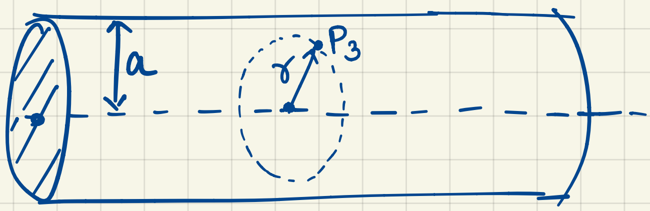
<u>Case II</u>:- $r = a$ at point P_2

Now, to find the magnetic field intensity at point P_2 on the surface of the wire. Make a circular loop of radius $(r = a)$.

$\therefore$ Similarly like (I)st we'll get $\rightarrow$ $\boxed{B = \frac{\mu_0 I}{2\pi a}}$

<u>Case III</u>:- $r < a$, at point P_2

To find the magnetic field intensity at point P_3 inside the surface of the cylindrical wire $\longrightarrow$ make a circular loop made of radius r $(r < a)$



Now, in this case the enclosed current I_e is not I but less than the value. Since the current distribution is uniform, the current enclosed is,

$$I_e = \frac{I r^2}{a^2}$$

Using Ampere's law, $\oint \vec{B}\cdot\vec{dl} = \mu_0 I_e$

$$\Rightarrow \oint B\,dl = \mu_0 \frac{I r^2}{a^2}$$

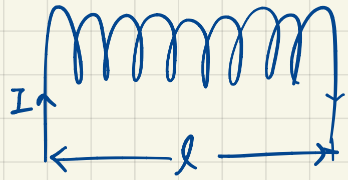
$$\Rightarrow B \oint dl = \mu_0 \frac{I r^2}{a^2}$$

$$\Rightarrow B(2\pi \cancel{r}) = \mu_0 \frac{I r^{\cancel{2}}}{a^2}$$

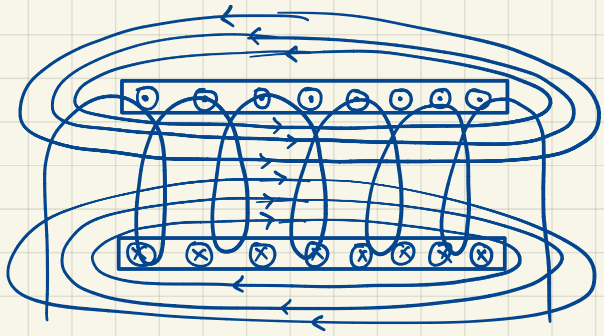
$$\Rightarrow B\,2\pi = \mu_0 \frac{I r}{a^2} \Rightarrow \boxed{B = \frac{\mu_0 I r}{2\pi a^2}}$$

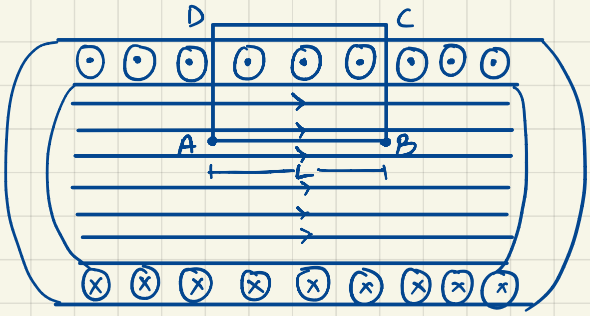
$B \propto r$

Magnetic Field due to Solenoid:



No. of turns = N





Let a solenoid consists of 'n' no. of turns per unit length and carry current I.
Magnetic field inside the solenoid is uniform and strong.
M.F. outside the solenoid is weak (almost zero)

Consider a close loop ABCD.

$$\therefore \oint B\cdot dl = \int_{AB} \vec{B}\cdot d\vec{l} + \int_{BC} \vec{B}\cdot d\vec{l} + \int_{CD} \vec{B}\cdot d\vec{l} + \int_{DA} \vec{B}\cdot d\vec{l}$$

Here, $\int_{CD} B\cdot dl = 0$ [B outside = 0]

$$\int_{BC} \vec{B} d\vec{l} = \int_{AD} \vec{B}\cdot d\vec{l} = 0 \quad [\because \vec{B} \perp d\vec{l}]$$

Hence,

$$\oint \vec{B}\cdot d\vec{l} = \oint_{AB} \vec{B}\cdot d\vec{l} + 0 + 0 + 0$$

$$\oint \vec{B}\cdot d\vec{l} = \int_{AB} \vec{B}\, d\vec{l} \cos 0^\circ$$

$$\oint \vec{B}\cdot d\vec{l} = \vec{B} \int_{AB} d\vec{l}$$

$$\boxed{\oint \vec{B}\cdot d\vec{l} = \vec{B}(L)} \text{ --- (I)}$$

According to Ampere's law :

$$\oint B\cdot dl = \mu_0 I$$

Here, N number of turns, are present

$$\therefore \oint B\cdot dl = \mu_0 N I$$

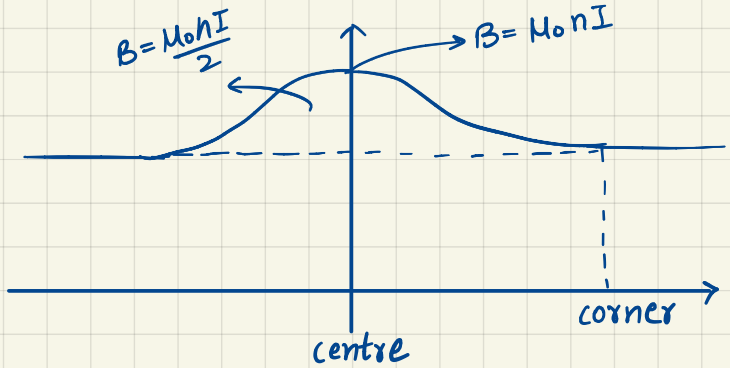
$$\Rightarrow B(L) = \mu_0 N I \text{ --- from (I)}$$

$$B = \frac{\mu_0 N I}{L}$$

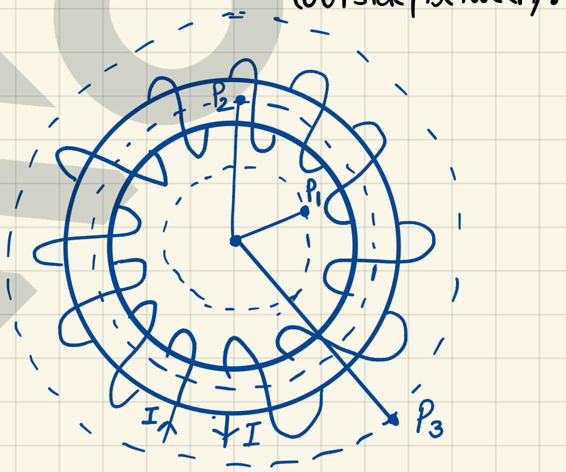
$$\boxed{\vec{B} = \mu_0 n I}$$

where, n = no. of turns per unit length ie., $\boxed{n = \frac{N}{L}}$

Graph:



Using Ampere's circuital Law, obtained the magnetic field inside a Toroid (Outside/Between):-



Case I) Inside

from Ampere's law:-

$$\oint \vec{B}\,d\vec{l} = \mu_0 I_{in} \quad [at\ P_1]$$

here, $\boxed{I_{in} = 0}$

$$\oint \vec{B}\,d\vec{l} = 0 \Rightarrow \boxed{B = 0}$$

Case II) Between the turns:-

from Ampere's law:- $\oint \vec{B}\,d\vec{l} = \mu_0 I_{in}$ (at P_2)

$$\oint B\,dl \cos\theta = \mu_0 I_{in}$$

$$B \oint dl = \mu_0 N I$$

$$B(2\pi r) = \mu_0 N I$$

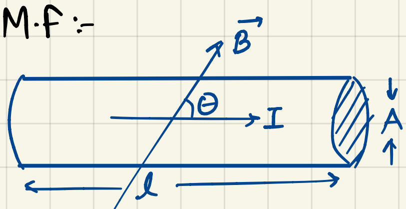
$$\boxed{B = \frac{\mu_0 N I}{2\pi r}} \quad \text{or} \quad \boxed{B = \mu_0 n I} \quad \left[\because n = \frac{N}{l} = \frac{N}{2\pi r}\right]$$

Case III) Outside:- (at P_3)

$$\oint B\,dl = \mu_0 I_{in}$$

$$\boxed{B = 0}$$

Force acting on a current carrying conductor placed in M.F:-



Consider a conductor of length l and area of section A carrying current I placed in a magnetic field at an angle θ as shown. If number density of electrons in the the conductor is n, then total no. of electrons in the conductor is: Aln.

As the force acting one electron is $f = e V_d B \sin\theta$ where V_d is the drift velocity of electrons.

So the total force acting on the conductor is $= Al n f$

$$= Al n (e V_d B \sin\theta)$$
$$= (A n e V_d) l B \sin\theta$$
$$\boxed{F = I l B \sin\theta}$$

Hence Proved

direction can be determined by Fleming's left hand rule.

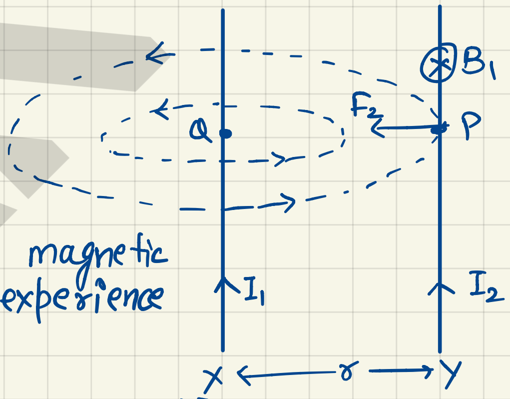
Force between two parallel straight conductors carrying current:

Consider two infinite long straight conductors (X and Y) carrying currents I_1 and I_2 in the same direction.

They are held parallel to each other at a distance 'r'.

Since magnetic field is produced due to current through each conductor, therfore each conductor experiences a force.

and, the force will be $I l B \sin\theta$.



Now, magnetic field at P due to current I_1

$$B_1 = \frac{\mu_0 I_1}{2\pi r} \quad \text{—(I)}$$

As the current carrying conductor Y lies in the magnetic field B_1, therefore the unit length of Y will experience a force given by–

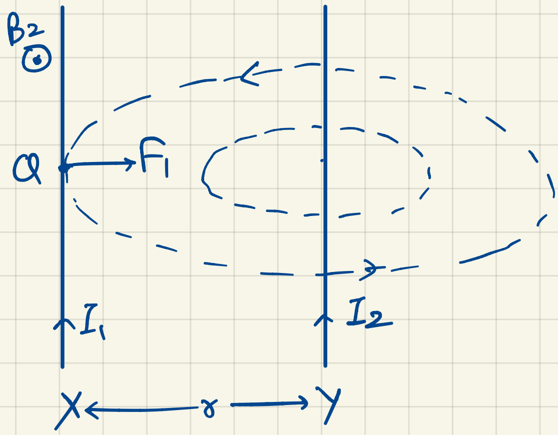
$$\Rightarrow F_2 = I_2 (1) B_1 \sin 90^\circ \quad [\because l = 1 \text{ (unit length)}]$$

$$F_2 = B_1 I_2 \times 1 \qquad (\because \sin 90 = 1)$$

$$\boxed{F_2 = \frac{\mu_0 I_1 I_2}{2\pi r}}$$

Magnetic field due to current I_2 at point Q

$$B_2 = \frac{\mu_0 I_2}{2\pi r} \quad \text{—(II)}$$



Similarly conductor X will also experience a force F_1 due to I_2 current.

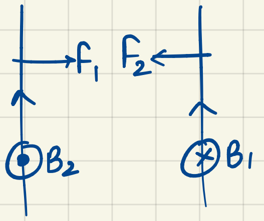
$$F_1 = B_2 I_1 \sin\theta \quad [l = 1 \text{ (unit length)}]$$

$$F_1 = B_2 I_1 \sin 90^\circ$$

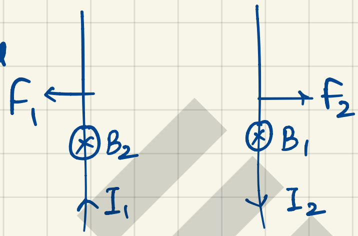
$$\boxed{F_1 = \frac{\mu_0 I_1 I_2}{2\pi r}}$$

We can observe that F_1 acts perpendicular to X and directed towards Y. Hence X and Y attract each other.

So, $F_1 = F_2$

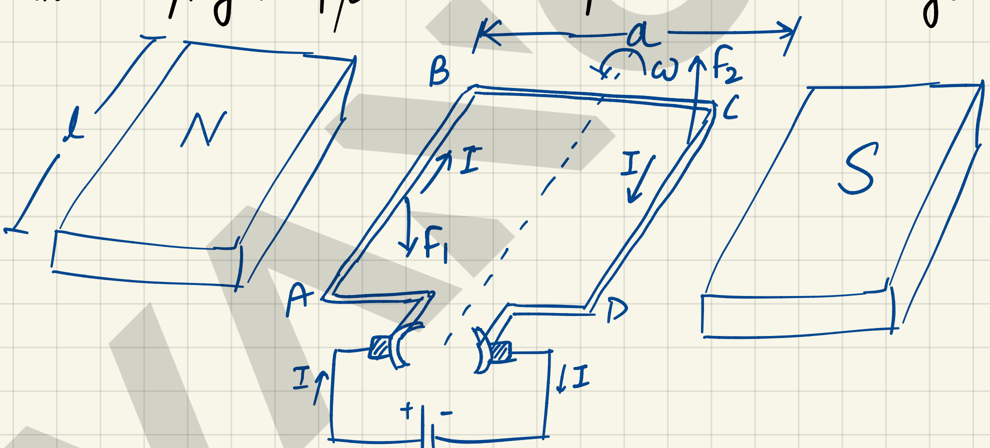


but when current will be in opposite directions, the conductors will repel each other and magnitude will be same as derived above



Hence
- Same current direction ⟶ attraction
- opposite current dirxⁿ ⟶ repulsion

TORQUE acting on a current carrying loop/coil in uniform M.f. (rectangular)



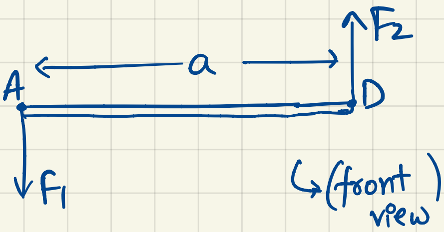
When a rectangular current carrying coil is placed in a uniform magnetic field then it experiences a torque. It does not experience a '<u>net</u>' <u>force</u>.

∴ Magnetic force on a current carrying conductor.

$$F = I l b \sin\theta \longrightarrow \text{on arms AB \& CD only}$$

but field exerts no force on the two arms AD and BC of loop because B is antiparallel to I.

Now, The magnetic field is perpendicular to the arm AB of the loop and exerts a force F_1 on it, which is directed into the plane of the loop.



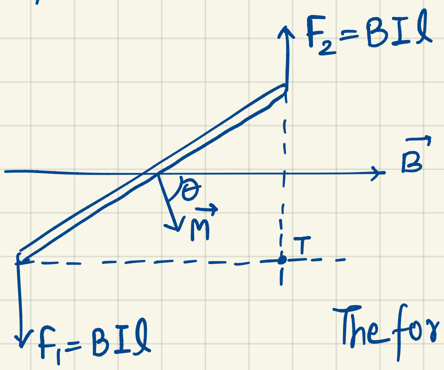
$$F_1 = I l B \sin 90° \Rightarrow \underline{I l B}$$

Similarly, the magnetic field exerts a force F_2 on arm CD, which is directed out of the plane of the loop.

$$F_2 = I l B = F_1$$

Thus, the '<u>net</u>' <u>force</u> on the loop is zero (as said earlier)

But, as we can see there will a torque on the loop due to the pair of forces F_1 and F_2



Now, consider the case when the plane of the loop, is not along the magnetic field and makes an angle with it.

Let the angle between the field and the normal to the coil be angle θ.

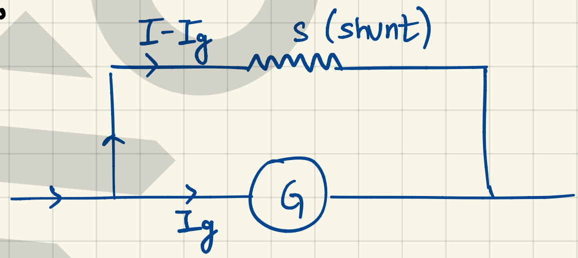
The force on arms AB and CD are F_1 and F_2

$$F_1 = F_2 = IBl \quad \text{— (I)}$$

$$M \propto IA$$
$$M = KIA \quad (\text{Here } k = 1)$$

$\Rightarrow \boxed{M = IA}$ → for N no. of turns, $\vec{M} = NI\vec{A}$

Conversion of Galvanometer into Ammeter:



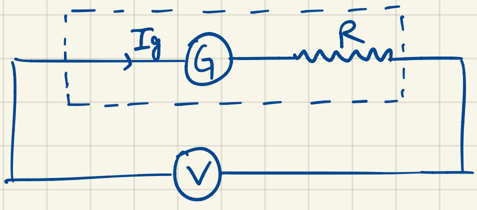
Galvanometer can be converted into ammeter by connecting a small Resistance S(shunt) in parallel with the galvanometer

I_g = max Current through galvanometer
I = ammeter range
R_g = Galvanometer Resistance

As S and G are connected in parallel, $S(I - I_g) = I_g R_g$

$$\boxed{S = \frac{I_g R_g}{I - I_g}}$$

Conversion of Galvanometer into Voltmeter:-



Galvanometer can be converted into voltmeter by connecting high resistance in series.

I_g = current through galvanometer
R = high resistance
V = External potential
R_g = Galvanometer resistance

total resistance = $R + R_g$

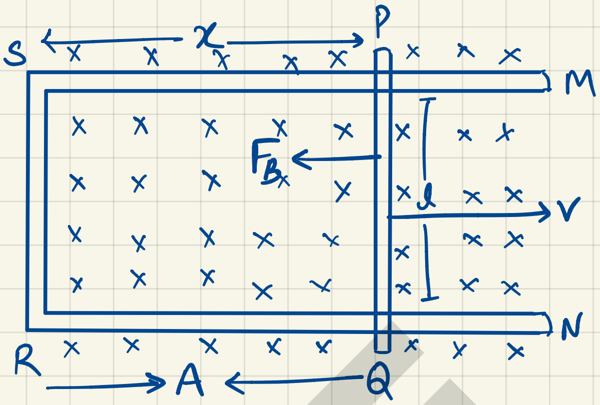
Now, acc. to ohm's law, $V = I_g(R_g + R)$

$$\frac{V}{I_g} = R + R_g$$

$$\boxed{R = \frac{V}{I_g} - R_g}$$

CHAPTER #6: Electromagnetic Induction

Motional EMF or Induced EMF:



Consider a rectangular conducting loop PQRS in the plane of the paper in which the conductor PQ is free to move.

Let the rod PQ is moved towards right with a constant velocity 'v', assume there is no loss of energy due to friction.

Let PQ is moved 'x' distance towards right, the area enclosed by loop PQRS increases. ⇒ Area (A) = lx
Therefore, the amount of magnetic flux linked with the loop increases. An emf is induced in the loop.

flux through area (A) ⇒ $\vec{B} \cdot \vec{A}$

$$\phi = BA\cos\theta$$
$$\phi = Blx\cos 0^\circ$$
$$\phi = Blx \quad \text{— (I)}$$

∴ Induced EMF in the coil is →

$$\mathcal{E} = \frac{-d\phi}{dt}$$
$$\mathcal{E} = \frac{-d}{dt} Blx \quad [\text{from (I)}]$$
$$\mathcal{E} = -Bl\left(\frac{dx}{dt}\right)$$
$$\boxed{\mathcal{E} = -Bl\,v}$$

Hence Proved

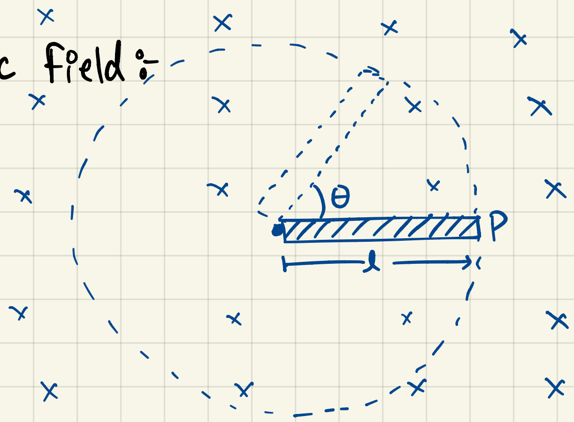
[∵ $\frac{dx}{dt}$ means rate of change of displacement which is velocity]

Force on the wire (external)

$$F = BIl \sin 90^\circ$$
$$F = B\left(\frac{Blv}{R}\right)l$$
$$\boxed{F = \frac{B^2 l^2 v}{R}}$$

[∵ $I = \frac{\mathcal{E}}{R} = \frac{Blv}{R}$]

Induced EMF due to rotation of Rod in Magnetic Field:-



Consider a metallic rod of length 'l' is placed in a uniform magnetic field, as shown in the figure,

Area covered by the rod on rotating by 2π angle $= \pi l^2$

$\therefore$ for 1 unit rotation (area) $= \frac{\cancel{\pi} l^2}{2\cancel{\pi}} = \frac{l^2}{2}$ (simple unitary method applied)

$\therefore$ for θ angle rotation $= \frac{l^2\theta}{2}$

$\therefore$ Area will be, $A = \frac{l^2\theta}{2}$ — (I)

Now, flux through area A, $\phi = \vec{B}\cdot\vec{A}$

$$\phi = BA\cos 0^\circ$$

$$\phi = B\left(\frac{l^2\theta}{2}\right)$$

Induced emf in the rod, $\varepsilon = -\frac{d\phi}{dt}$

$$\varepsilon = -\frac{d}{dt}\left(B\frac{l^2\theta}{2}\right)$$

$$\varepsilon = \frac{Bl^2}{2}\frac{d\theta}{dt}$$

$$\boxed{\varepsilon = \frac{Bl^2\omega}{2}}$$

where, ω = angular velocity $\left(\frac{d\theta}{dt}\right)$

Hence Proved

Self-Induction of Solenoid:

Consider a solenoid having 'N' turns with length l and cross-section area A. I is the current flowing through it. So, there will be a magnetic field at a given point in the solenoid, represent it by B.

Now, The magnetic flux per turn will be equal to product of B and area of each turn.

$$= \frac{\mu_0 N I}{l} \times A$$

$\therefore$ Total magnetic flux will be given by product of flux present in each turn and the no. of turns.

$$\phi = \frac{\mu_0 N I}{l} \times N$$

$$\phi = \frac{\mu_0 N^2 I}{l} \quad \text{— (I)}$$

And, we also know, $\phi = LI$ — (II)

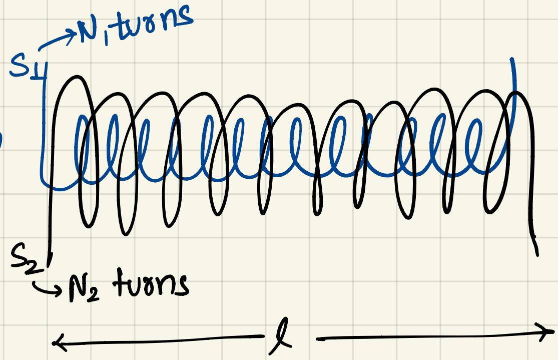
$\therefore$ from (I) & (II) $\quad L\cancel{I} = \frac{\mu_0 N^2 \cancel{I}}{l}$

$$\boxed{L = \frac{\mu_0 N^2}{l}}$$

Hence Proved

This is self-inductance of a solenoid.

Mutual inductance of two solenoids:-



Consider two long solenoids S_1 and S_2 each of length l. N_1 and N_2 are the no. of turns in the solenoid S_1 and S_2 respectively.

S_2 is wound closely over S_1, so both the solenoids are considered to have the same area of cross section 'A'.

I_1 is the current flowing through S_1.

Now, the magnetic field B_1 produced at any point inside solenoid S_1 due to current I_1 is

$$B_1 = \frac{\mu_0 N_1 I_1}{l} \quad \text{— (I)}$$

And, the magnetic flux linked with each turn of S_2 is equal to $B_1 A$.

Total magnetic flux linked with solenoid S_2 having N_2 turns is

$$\phi_2 = B_1 A N_2$$

$$\Rightarrow \phi_2 = \left(\frac{\mu_0 N_1 I_1}{l}\right) A N_2 \qquad [\text{from (I)}]$$

$$\Rightarrow \phi_2 = \left(\frac{\mu_0 N_1 N_2 I_1}{l}\right) A \quad \text{— (II)}$$

but $\quad \phi_2 = M I_1 \quad$ — (III) , where M is the coefficient of mutual induction between S_1 and S_2

$\therefore$ from (II) & (III) $\Rightarrow$ $M I_1 = \left(\frac{\mu_0 N_1 N_2 I_1}{l}\right) A$

$$\therefore \boxed{M = \frac{\mu_0 N_1 N_2 A}{l}}$$

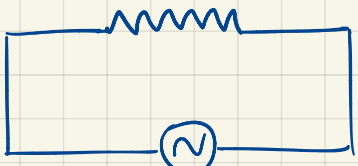
Hence Proved

And, if the core is filled with a magnetic material of permeability μ

$$\boxed{M = \frac{\mu N_1 N_2 A}{l}}$$

CHAPTER #7 : Alternating Current

AC Voltage applied to a Resistance :



$E = E_0 \sin\omega t$

Consider a resistor ~~of resistor~~ of resistance R is connected in series with a circuit containing Alternating EMF → $E_0 \sin\omega t$ —(I)

∵ current through the circuit,

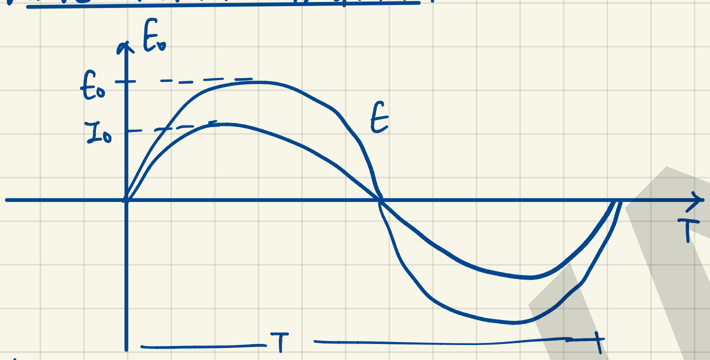
$$I = \frac{E}{R} \Rightarrow I = \frac{E_0 \sin\omega t}{R}$$

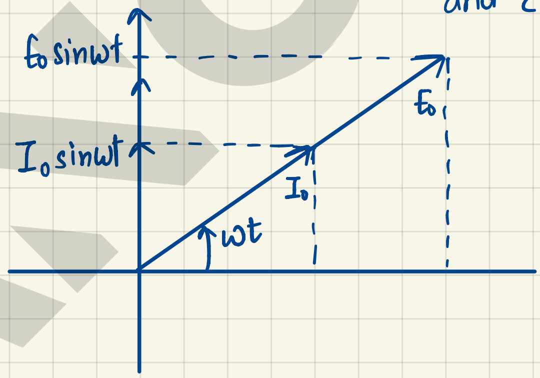
$$\boxed{I = I_0 \sin\omega t} \text{—(II)}$$

Hence Proved

comparing (I) & (II) we can say that, there is no phase difference between current and EMF.

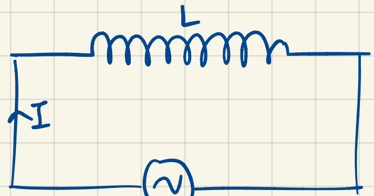
WAVE FORM DIAGRAM :-





Phasor diagram.

AC voltage applied to an Inductor :



$E = E_0 \sin\omega t$

Consider an inductor of inductance 'L' connected in series with a circuit containing Alternating EMF → $E_0 \sin\omega t$ —(I)

An EMF will induce in the inductor due to the current I.

i.e. $e = -L \dfrac{dI}{dt}$

According to lenz law, the induced emf will oppose the alternating emf. we can say,

$$E = -e$$

$$\Rightarrow E = -\left(-L \frac{dI}{dt}\right)$$

$$\Rightarrow E = L \frac{dI}{dt}$$

$$\Rightarrow dI = \frac{E}{L} dt$$

$$\Rightarrow dI = \frac{E_0 \sin\omega t}{L} dt$$

for total current integrating both side,

$$\int dI = \int \frac{\mathcal{E}_0}{L} \sin\omega t \, dt$$

$$\Rightarrow I = \frac{\mathcal{E}_0}{L}\left(-\frac{\cos\omega t}{\omega}\right)$$

$$\Rightarrow I = \frac{-\mathcal{E}_0}{\omega L} \cos\omega t$$

$$\Rightarrow I = -\frac{\mathcal{E}_0}{\omega L}\left(\sin\left(\frac{\pi}{2} - \omega t\right)\right)$$

$$\Rightarrow I = \frac{\mathcal{E}_0}{\omega L} \sin\left(\omega t - \frac{\pi}{2}\right) \quad \text{— (I)}$$

$\left[\because \sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta \quad \sin(-\theta) = -\sin\theta\right]$

when $\sin\left(\omega t - \frac{\pi}{2}\right)$ will be 1, the I will be peak value. i.e. $I_0 = \frac{\mathcal{E}_0}{\omega L}$

$$\text{(I)} \Rightarrow \boxed{I = I_0 \sin\left(\omega t - \frac{\pi}{2}\right)} \quad \text{— (II)}$$

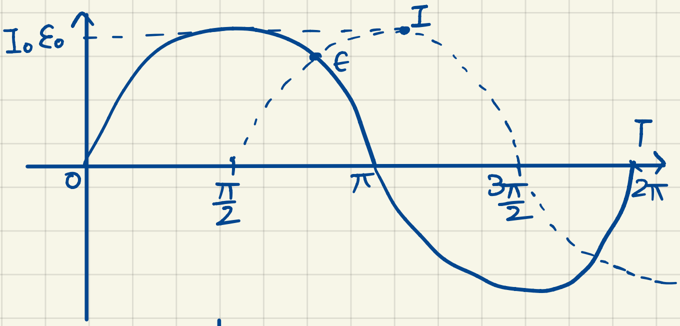
Hence Proved

on comparing (I) & (II), we see that I and $\mathcal{E}$ have different phase

i.e. phase difference between I and $\mathcal{E}$.

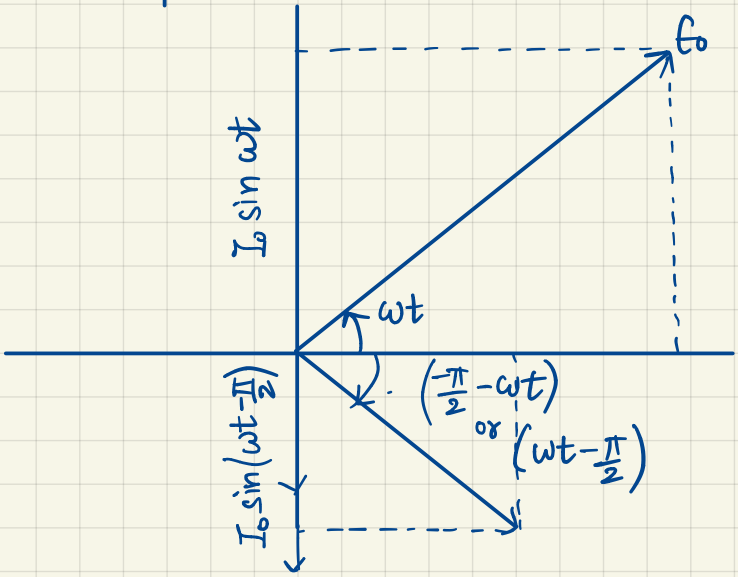
$$\phi = \cancel{\omega t} - \cancel{\omega t} + \frac{\pi}{2}$$

$$\Rightarrow \phi = \frac{\pi}{2} \quad \therefore \text{voltage leads current.}$$



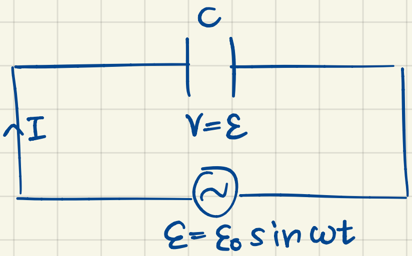
Wave form diagram for I and $\mathcal{E}$

Phasor diagram for I and $\mathcal{E}$



AC voltage applied to a capacitor:

Consider a capacitor of capacitance 'C' is connected in series containg AC of emf of $\mathcal{E}_0 \sin\omega t$



$$\mathcal{E} = \mathcal{E}_0 \sin\omega t \quad \text{— (I)}$$

The maximum voltage of the capacitor will be equal to emf of the AC.

Also, charge on capacitor, $q = CV$

$$q = C\mathcal{E}_0 \sin\omega t \quad [\because V = \mathcal{E}]$$

Instantaneous current in the circuit,

$$I = \frac{dq}{dt}$$

$$I = \frac{d}{dt}(C\mathcal{E}_0 \sin\omega t)$$

$$\Rightarrow I = C\mathcal{E}_0 \frac{d}{dt}(\sin\omega t)$$

$$\Rightarrow I = C\mathcal{E}_0\, \omega \cos\omega t$$

$$\Rightarrow I = C\mathcal{E}_0\, \omega \left(\sin\frac{\pi}{2} + \omega t\right) \quad \text{— (II)}$$

Now, I will be max(peak) when $\left(\sin\frac{\pi}{2} + \omega t\right)$ will become 1.

$$\therefore \quad I_0 = C\mathcal{E}_0 \omega$$

So, (II) $\Rightarrow$ $\boxed{I = I_0 \sin\left(\omega t + \frac{\pi}{2}\right)}$ — (III)

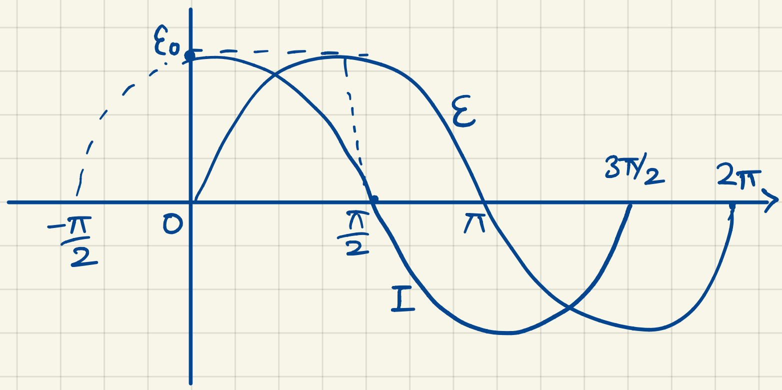
Hence Proved

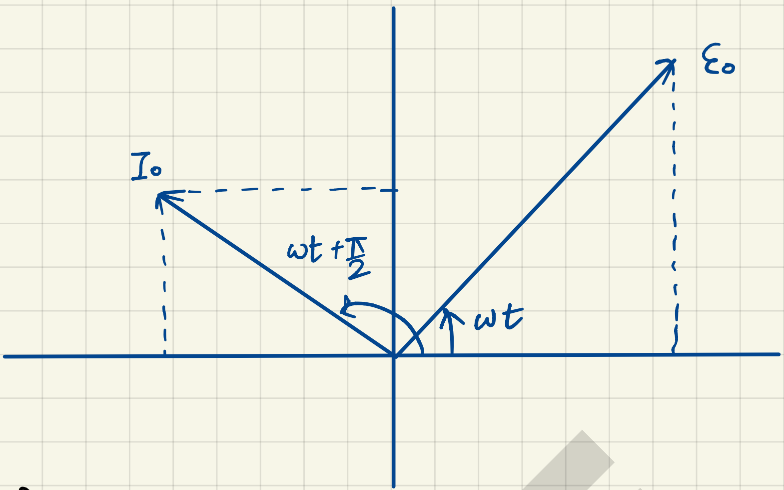
Comparing (I) and (III), we see that there is a phase difference between I and $\mathcal{E}$ due to which current is leading behind the voltage.

$\therefore$ Phase difference between I and $\mathcal{E}$, $\phi = \cancel{\omega t} + \frac{\pi}{2} - \cancel{\omega t}$

$$\phi = \frac{\pi}{2}$$

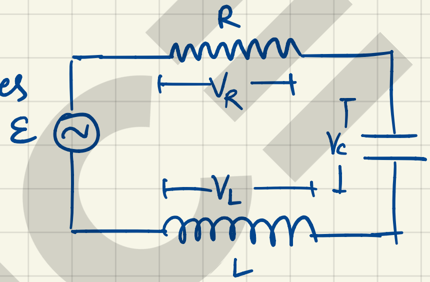
Wave form diagram for I and $\mathcal{E}$





Phasor diagram for I and $\mathcal{E}$:-

Impedence in series LCR circuit :-

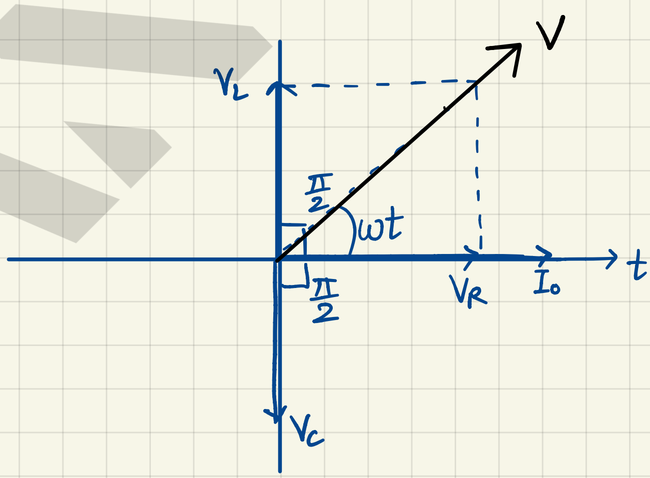


Consider a LCR circuit connected to an AC source in series

Here, voltage drop across resistance, capacitor and inductor is -

$$\left.\begin{aligned} V_R &= I\,R \\ V_c &= I\,X_c \\ V_L &= I\,X_L \end{aligned}\right\} \text{(I)} \quad \left[\begin{aligned} &\text{where,} \\ &X_c = \omega L \\ &X_L = \frac{1}{\omega L} \end{aligned}\right]$$

Phasor diagram for L, C, R circuit :-



Consider, $\mathcal{E}_0$ is the total voltage supplied in the circuit.

In the above phasor diagram, let $V_L > V_c$.

$$\therefore \; V_{Lc} = (V_L - V_c) \quad \text{— (II)}$$

Now, voltage across all the components,

$$V = \sqrt{(V_L - V_c)^2 + V_R^2}$$

$$V = \sqrt{(IX_L - IX_c)^2 + (IR)^2}$$

$$V = \sqrt{I^2\{(X_L - X_c)^2 + R^2\}}$$

$$V = I\sqrt{\{(X_L - X_c)^2 + R^2\}}$$

$$\frac{V}{I} = \sqrt{\{(X_L - X_c)^2 + R^2\}}$$

$$\boxed{Z = \sqrt{(X_L - X_c)^2 + R^2}}$$

Z is called impedance

Hence Proved

Resonating frequency in series LCR circuit :-

Resonance occurs when inductive reactance becomes equal to capacitive reactance.

$$X_L = X_C$$
$$\omega L = \frac{1}{\omega C}$$
$$\omega^2 = \frac{1}{LC}$$
$$\omega = \frac{1}{\sqrt{LC}}$$
$$2\pi\nu = \frac{1}{\sqrt{LC}}$$
$$\boxed{\nu = \frac{1}{2\pi\sqrt{LC}}}$$

Hence Proved

At ν, X_L will become equal to X_C and resonance will occur, and the frequency is known as resonating frequency.

Average Power in LCR Circuit :

We know that a voltage $\mathcal{E} = \mathcal{E}_0 \sin\omega t$ applied to a series RLC circuit drives a current in the circuit given by

$i = i_0 \sin(\omega t - \phi)$; where $i_0 = \frac{V_0}{Z}$ &

$\phi = \tan^{-1}\left(\frac{X_C - X_L}{R}\right)$

$\therefore$ Instantaneous power by the source is:

$$P = \mathcal{E}I \Rightarrow \mathcal{E}_0 \sin\omega t \times i_0 \sin(\omega t - \phi)$$

$$P = \frac{\mathcal{E}_0 i_0}{2}\left[\cos\phi - \cos(2\omega t + \phi)\right] \longrightarrow \text{(I)}$$

Now, the average power over a cycle is given by the average of the two terms in R.H.S. of the above equation.

But we can see that only the second term is time dependent. $\therefore$ Its average will be zero ($\because$ positive half of the cosine cancels the negative second half)

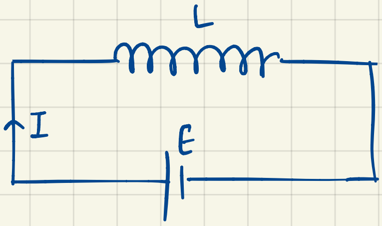
$$\therefore \quad P = \frac{\mathcal{E}_0 i_0}{2}\cos\phi$$

$$P = \left(\frac{\mathcal{E}_0}{\sqrt{2}}\right)\left(\frac{i_0}{\sqrt{2}}\right)\cos\phi \qquad \left[\because \frac{1}{2} = \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}}\right]$$

$$\boxed{P = E_{rms} I_{rms} \cos\phi}$$

Hence Proved.

Energy stored in an Inductor:



Consider an inductor of inductance L connected to a voltage source E as shown in figure.

As we know, $P = EI$

$$P = LI\frac{dI}{dt} \qquad \left[\because E = \frac{dI}{dt}\right]$$

$$\frac{dW}{\cancel{dt}} = LI\frac{dI}{\cancel{dt}} \qquad \left[\because P = \frac{dw}{dt}\right]$$

$$dw = LI\,dI \quad \text{—— (I)}$$

Integrating both sides, $\int dw = \int_0^{I_0} LI\,dI$ $\qquad$ (I_0 = max current in the circuit)

$$W = L\int_0^{I_0} I\,dI$$

$$W = L\left[\frac{I^2}{2}\right]_0^{I_0}$$

$$W = L\left[\frac{I_0^2}{2} - 0\right]$$

$$W = \frac{1}{2}LI_0^2$$

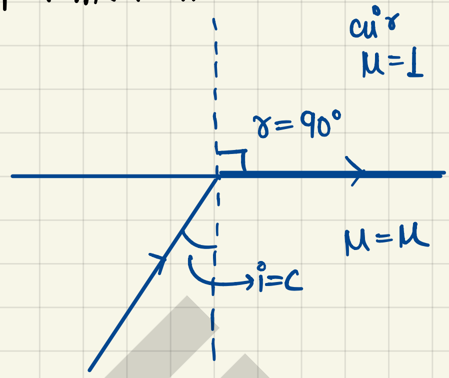
This work is stored in the circuit as magnetic potential energy.

$\therefore$ $$\boxed{U = \frac{1}{2}LI_0^2}$$

Hence Proved

Relation between Critical angle and refractive index of a medium:

Consider a light ray travelling from denser medium (μ) to rarer (1).



According to snell's law: $\mu \sin C = (1) \sin 90°$

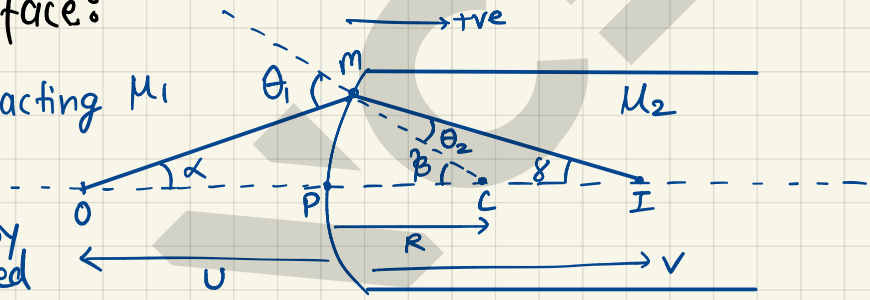
$$\mu \sin C = 1$$

$$\boxed{\mu = \frac{1}{\sin C}}$$

Hence Proved

Refraction at a spherical surface:

Figure shows refraction by convex refracting surface.



Let α, β and γ be the angle made by incident ray, normal and refracted ray with the principle axis.

The normal drawn from the convex refracting surface passes through the centre of curvature (c). All distances are measured from pole and, the direction of incident ray is taken +ve.

Now, In ΔOMC,
$$\theta_1 = \alpha + \beta$$
; In ΔCMI,
$$\beta = \theta_2 + \gamma$$
$$\theta_2 = \beta - \gamma$$

Now, By snell's law:

$$\mu_1 \sin\theta_1 = \mu_2 \sin\theta_2$$

as θ_1 & θ_2 are very small, $\therefore$ $\sin\theta_1 \approx \theta_1$ and $\sin\theta_2 \approx \theta_2$

$$\Rightarrow \quad \mu_1\theta_1 = \mu_2\theta_2$$

$$\mu_1(\alpha+\beta) = \mu_2(\beta-\gamma) \quad \text{--- (I)}$$

Here, α, β & γ are very small

$$\therefore \tan\alpha = \frac{h}{-U} \approx \alpha$$

$$\tan\beta = \frac{h}{R} \approx \beta$$

$$\tan\gamma = \frac{h}{V} \approx \gamma$$

$$\therefore \text{(I)} \Rightarrow \quad \mu_1\left(-\frac{h}{U} + \frac{h}{R}\right) = \mu_2\left(\frac{h}{R} - \frac{h}{V}\right)$$

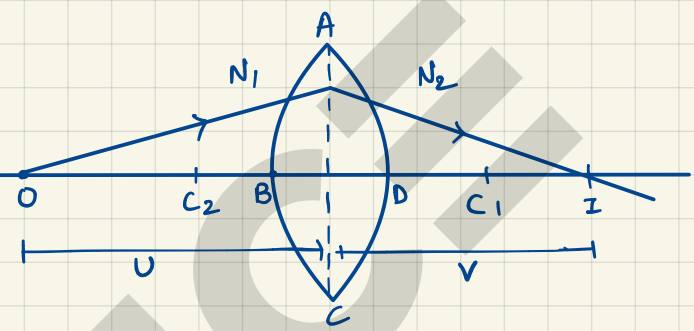
$$\Rightarrow \mu_1\left(\frac{1}{R}-\frac{1}{u}\right) = \mu_2\left(\frac{1}{R}-\frac{1}{v}\right)$$

$$\Rightarrow \frac{\mu_1}{R} - \frac{\mu_1}{u} = \frac{\mu_2}{R} - \frac{\mu_2}{v}$$

$$\Rightarrow \boxed{\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2-\mu_1}{R}}$$

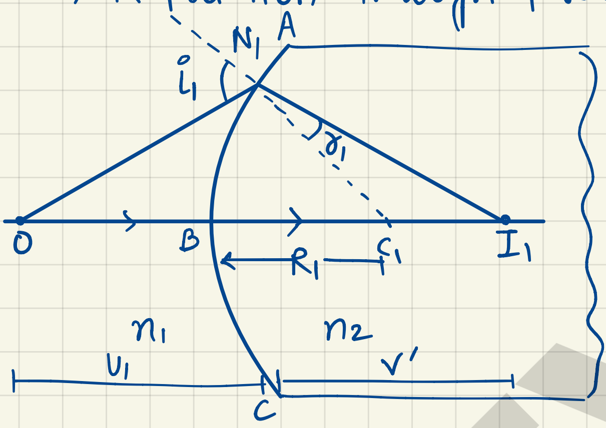
Hence Proved

Lens Maker Formula :



Consider a convex lens (thick), Let an object is placed on the principle axis at 'O'. The image formed by the convex thick lens is at I.

I) Refraction through first surface (ABC):

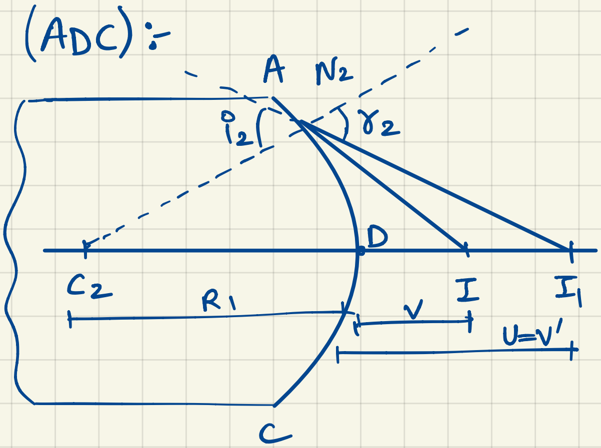


If surface ADC is not present then image will be formed at I_1 as shown in the figure.

$\therefore$ According to refraction formula:

$$\Rightarrow \frac{n_2-n_1}{R_1} = \frac{n_2}{v'} - \frac{n_1}{u_1} \quad \text{—— (I)}$$

II) Further, refraction through second surface (ADC):-



If the surface ABC is not present then image I_1 will behave like object and the image by second surface will be formed at I as shown in figure.

Now, According to refraction formula,

$$\frac{n_1-n_2}{R_2} = \frac{n_1}{v} - \frac{n_2}{v'} \quad \text{—— (II)}$$

Adding (I) & (II) :-
$$\frac{n_2-n_1}{R_1} + \frac{n_1-n_2}{R_2} = \cancel{\frac{n_2}{v'}} - \frac{n_1}{u_1} + \frac{n_1}{v} - \cancel{\frac{n_2}{v'}}$$

$$\Rightarrow \frac{n_2-n_1}{R_1} + \frac{n_1-n_2}{R_2} = \frac{n_1}{v} - \frac{n_1}{u_1}$$

$$\Rightarrow \frac{n_2-n_1}{R_1} - \frac{(n_2-n_1)}{R_2} = n_1\left(\frac{1}{v}-\frac{1}{u_1}\right)$$

$$\Rightarrow (n_2 - n_1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = n_1\left(\frac{1}{f}\right)$$

$$\Rightarrow \frac{n_2 - n_1}{n_1}\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{1}{f}$$

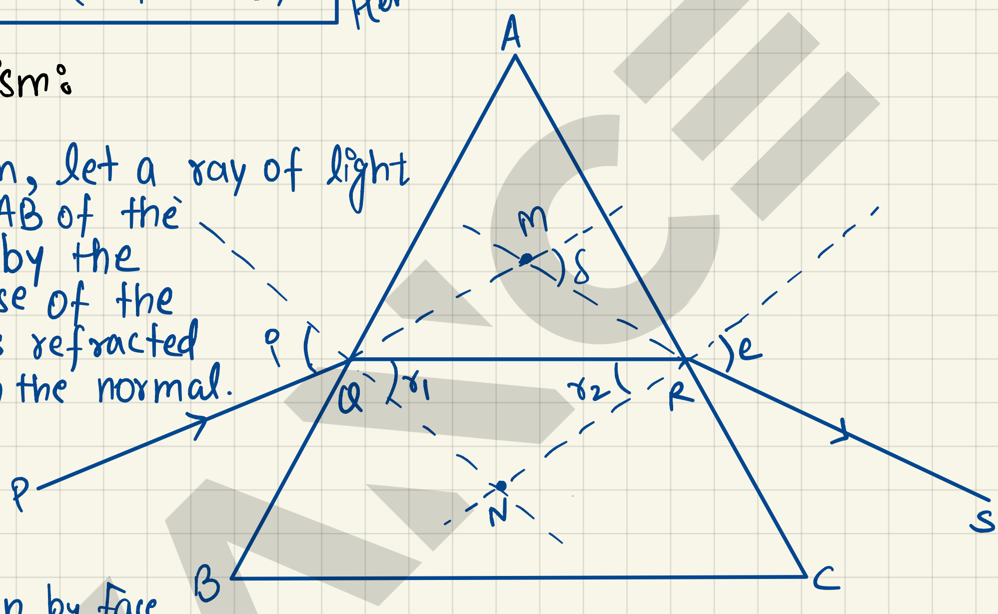
$$\Rightarrow \frac{n_2}{n_1} - 1\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{1}{f}$$

$$\Rightarrow \boxed{\frac{1}{f} = (n_2 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}$$

Hence Proved

Refraction through Prism:

Consider a triangular prism, let a ray of light PQ strikes on the face AB of the prism and then refracted by the face AB towards the base of the prism BC and again QR is refracted by the fase AC away from the normal.



i = Angle of incidence

r_1 and r_2 = angle of refraction by face AB and AC respectively

A = angle of prism

δ = angle of deviation

In ΔQNR, $\angle r_1 + \angle r_2 + \angle QNR = 180^\circ$ — (I)

In quadrilateral AQNR, $\angle A + 90^\circ + \angle QNR + 90^\circ = 360^\circ$

$$\Rightarrow \angle A + \angle QNR = 360^\circ - 180^\circ$$

$$\Rightarrow \angle A + \angle QNR = 180^\circ \quad \text{— (II)}$$

from (I) & (II) $\Rightarrow \angle r_1 + \angle r_2 + \cancel{\angle QNR} = \angle A + \cancel{\angle QNR}$

$$\angle r_1 + \angle r_2 = \angle A$$

or $\boxed{A = r_1 + r_2}$ — (III)

Also, $\delta = \delta_1 + \delta_2$

$$\Rightarrow \delta = (i - r_1) + (e - r_2)$$

$$\Rightarrow \delta = (i + e) - (r_1 + r_2)$$

$\Rightarrow \quad \delta = (i+e) - A \qquad$ (from eq (III))

or $\boxed{i + e = \delta + A}$ —— (IV)

when, $\delta = \delta_{min}$, then $i = e$

$r_1 = r_2 = r$

$\therefore$ eq (II) becomes, $r + r = A$

$2r = A$

$\boxed{r = \frac{A}{2}}$ —— (V)

And, eq (IV) becomes $\Rightarrow \quad i + i = \delta_m + A$

$2i = \delta_m + A$

$\boxed{i = \frac{\delta_m + A}{2}}$ —— (VI)

Now, According to snell's law,

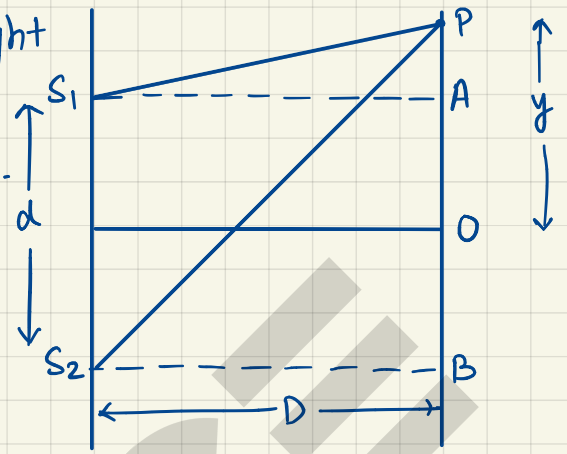
$\frac{\sin i}{\sin r} = \mu$ (where μ is refractive index of the material present in the prism)

$\Rightarrow \quad \frac{\sin\left(\frac{\delta_m + A}{2}\right)}{\sin\left(\frac{A}{2}\right)} = \frac{\mu_2}{\mu_1}$ (from (V) & (VI))

$$\boxed{\mu_{21} = \frac{\sin\left(\frac{\delta_m + A}{2}\right)}{\sin(A/2)}}$$

Hence Proved

Position and width of the fringe in interference:



The distance between any two consecutive bright fringe is equal to the width of a dark fringe and the distance between any two consecutive dark fringe is equal to the width of a bright fringe.

Consider light from two slit S_1 and S_2 superimposed at point P on the screen bright and dark fringes.

Let d be the distance between two slits S_1 and S_2 and D be the distance between slit and screen.

Now, at point P the path difference of two waves is:

$$\Delta x = S_2P - S_1P \quad \text{—— (I)}$$

In ΔS_1AP,

Pytha:

$$S_1P^2 = S_1A^2 + AP^2$$
$$= D^2 + \left(y - \frac{d}{2}\right)^2 \quad \text{—— (II)}$$

In ΔS_2BP,

$$S_2P^2 = S_2B^2 + BP^2$$
$$= D^2 + \left(y + \frac{d}{2}\right)^2 \quad \text{—— (III)}$$

Now, eq (III) − eq (II)

$$S_2P^2 - S_1P^2 = D^2 + \left(y+\frac{d}{2}\right)^2 - D^2 + \left(y-\frac{d}{2}\right)^2$$

$$(S_2P - S_1P)(S_2P + S_1P) = 4y \times \frac{d}{2}$$

$$(S_2P - S_1P)(S_2P + S_1P) = 2yd$$

Assuming P very close to O, such that $S_1P \approx S_2P = D$

$$\therefore (D + D)\Delta x = 2yd$$

$$\cancel{2}D\Delta x = \cancel{2}yd$$

$$\boxed{\Delta x = \frac{yd}{D}}$$

Case I) for Maxima,

$$\Delta x = n\lambda$$

$$\frac{yd}{D} = n\lambda$$

$$\boxed{y = \frac{n\lambda D}{d}}$$

when, n=0, y=0 (Central bright fringe)

n=1, $y = \frac{\lambda D}{d}$ (1st B.F.)

⋮

n=n, $y = \frac{n\lambda D}{d}$ (n^{th} B.F.)

Case II) for minima:

$$\Delta x = (2n-1)\lambda/2$$

$$\frac{yd}{D} = (2n-1)\lambda/2$$

$$\boxed{y = \frac{2(n-1)\lambda D}{2d}}$$

when, $n=1$, $y = \frac{\lambda D}{2d}$ [1st DF]

$n=2$, $y = \frac{3\lambda D}{2d}$ [2nd DF]

⋮

$n=n$, $y = \frac{(2n-1)\lambda D}{2d}$ (n^{th} D.F)

∴ Alternate Dark & Bright fringes appear.

Now, Expression for fringe width:-

The difference between 2 consecutive bright fringes gives the fringe width of dark fringes & bright fringes.

$$\beta_{dark} = y_{n+1} - y_n$$

$$= (n+1)\frac{\lambda D}{d} - \frac{n\lambda D}{d}$$

$$\boxed{\beta = \frac{\lambda D}{d}}$$

Similarly, for bright → $\boxed{\beta = \frac{\lambda D}{d}}$

CHAPTER #11: Dual Nature of Radiation and Matter

de-Broglie Equation:

For a radiation of frequency (ν) & wavelength (λ) propagating in vaccum, the energy of one photon is:

$$\boxed{E = h\nu} \quad \text{—— (I)}$$

According to Einstein mass-energy equivalence,

$$\boxed{E = mc^2} \quad \text{—— (II)}$$

Comparing (I) & (II), $h\nu = mc^2$

$$\boxed{m = \frac{h\nu}{c^2}} \quad \text{—— (III)}$$

Now, momentum of each photon is, $P = mc$

$$P = \frac{h\nu}{c^{\cancel{2}}} \times \cancel{c}$$

$$P = \frac{h\nu}{c}$$

$$P = \frac{h}{c/\nu}$$

$$P = \frac{h}{\lambda} \qquad \left[\because \lambda = \frac{c}{\nu}\right]$$

$$\boxed{\lambda = \frac{h}{P}} \longrightarrow \text{this is de-broglie eq}^n.$$

Let us take an example, $e^{\ominus}$ accelerated through a potential diff. 'V', then its K.E. can be written as:

$$\boxed{K = eV}$$

The linear momentum & K.E. of $e^{\ominus}$ moving with velocity 'v' are:-

$$\boxed{P = mv}$$

&

$$K = \frac{1}{2}mv^2$$

multiplying 'm' both sides: $mK = \frac{1}{2}m^2v^2$

$$2mK = m^2v^2$$

Square root both side: $\sqrt{2mK} = \sqrt{m^2v^2}$

$$\boxed{mv = \sqrt{2mK}}$$

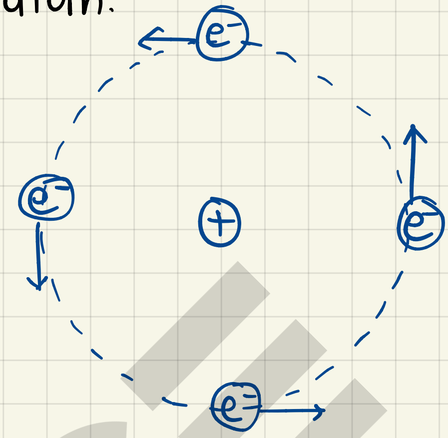
Now, from de-broglie eq^n:- $\lambda = \frac{h}{P}$

$$\lambda = \frac{h}{mv}$$

$$\boxed{\lambda = \frac{h}{\sqrt{2mK}}}$$

Substituting $\left.\begin{aligned} h &= 6.63 \times 10^{-34}\ Js \\ m &= 9.1 \times 10^{-31}\ kg \\ e &= 1.6 \times 10^{-19}\ C \end{aligned}\right\}$ $\boxed{\lambda = \frac{1.227}{\sqrt{V}}}$

Using Bohr's theory of Hydrogen atoms, derive the expression for total Energy of $e^{\ominus}$ in stationary states of the atom.



Consider an electron of mass m and charge e revolving with velocity v around a nucleus having atomic number z. Then the centripetal force required by the electron is provided by electrostatic force of attraction between nucleus and electron according to equation:

$$F_e = F_c$$

$$\frac{k q_1 q_2}{r^2} = \frac{mv^2}{r}$$

$$\Rightarrow \frac{k e z e}{r^2} = \frac{mv^2}{r}$$

$$\frac{k e z e}{r} = mv^2 \quad \text{— I}$$

Acc. to Bohr's Postulates : $mvr = \frac{nh}{2\pi}$ — II

Now, $\frac{\text{II}}{\text{I}}$:- $\frac{m^2 v^2 r^2}{mv^2} = \frac{n^2h^2}{4\pi^2} \times \frac{r}{kZe^2}$

$$mr = \frac{n^2h^2}{4\pi^2} \times \frac{4\pi\varepsilon_0}{Ze^2}$$

$$\boxed{r = \frac{n^2h^2\varepsilon_0}{m\pi Ze^2}} \quad \text{Hence Proved} \quad \text{— III}$$

Radius of n^{th} orbit

Now, Velocity of e^- in stationary orbits:

$$mvr = \frac{nh}{2\pi}$$

$$\frac{mv\, n^2h^2\varepsilon_0}{m\pi Ze^2} = \frac{nh}{2\pi}$$

$$\boxed{v = \frac{Ze^2}{2nh\varepsilon_0}} \quad \text{Hence Proved}$$

Velocity of $e^{\ominus}$ in n^{th} energy level

Now, Energy of $e^{\ominus}$ in stationary orbits:

(i) $K.E. = \frac{1}{2}mv^2$

$$= \frac{1}{2} m \left(\frac{Ze^2}{2nh\varepsilon_0}\right)^2$$

$$\boxed{KE = \frac{m\pi^2 e^4}{8n^2h^2\varepsilon_0^2}}$$

(ii) $PE = \frac{kq_1q_2}{r}$

$$= \frac{k(-e)\,(Ze)}{\frac{n^2h^2\varepsilon_0}{\pi m Ze^2}}$$

$$\boxed{PE = \frac{-2Z^2e^4m}{8n^2h^2\varepsilon_0^2}}$$

(iii) $TE = KE + PE$

$$\boxed{T.E. = \frac{-mz^2e^4}{8n^2h^2\varepsilon_0}}$$